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A possible solution to the problem of time in quantum cosmology

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ABSTRACT

We argue that in classical and quantum theories of gravity the configuration space and Hilbert space may not be constructible through any finite procedure. If this is the case then the "problem of time" in quantum cosmology may be a pseudo-problem, because the argument that time disappears from the theory depends on constructions that cannot be realized by any finite beings that live in the universe. We propose an alternative formulation of quantum cosmological theories in which it is only necessary to predict the amplitudes for any given state to evolve to a finite number of possible successor states. The space of accessible states of the system is then constructed as the universe evolves from any initial state. In this kind of formulation of quantum cosmology time and causality are built in at the fundamental level. An example of such a theory is the recent path integral formulation of quantum gravity of Markopoulou and Smolin, but there are a wide class of theories of this type.

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1 Introduction

The problem of time in quantum cosmology is one of the key conceptual problems faced by theoretical physics at the present time. Although it was first raised during the 1950's, it has resisted solution, despite many different kinds of attempts[1, 2, 3, 4, 5]. Here we would like to propose a new kind of approach to the problem. Basically, we will argue that the problem is not with time, but with some of the assumptions that lead to the conclusion that there is a problem. These are assumptions that are quite satisfactory in ordinary quantum mechanics, but that are problematic in quantum gravity, because they may not be realizable with any constructive procedure. In a quantum theory of cosmology this is a serious problem, because one wants any theoretical construction that we use to describe the universe to be something that can be realized in a finite time, by beings like ourselves that live in that universe. If the quantum theory of cosmology requires a non-constructible procedure to define its formal setting, it is something that could only be of use to a mythical creature of infinite capability. One of the things we would like to demand of a quantum theory of cosmology is that it not make any reference to anything at all that might be posited or imagined to exist outside the closed system which is the universe itself.

We believe that this requirement has a number of consequences for the problem of constructing quantum a good quantum theory of cosmology. These have been discussed in detail elsewhere [4, 6, 7]. Here we would like to describe one more implication of the requirement, which appears to bear on the problem of time.

We begin by summarizing briefly the argument that time is not present in a quantum theory of cosmology. In section 3 we introduce a worry that one of the assumptions of the argument may not be realizable by any finite procedure. (Whether this is actually the case is not known presently.) We explain how the argument for the disappearance of time would be affected by this circumstance. Then we explain how a quantum theory of cosmology might be made which overcomes the problem, but at the cost of introducing a notion of time and causality at a fundamental level. As an example we refer to recent work on the path integral for quantum gravity[8], but the form of the theory we propose is more general, and may apply to a wide class of theories beyond quantum general relativity.

2 The argument for the absence of time

The argument that time is not a fundamental aspect of the world goes like this[1]. In classical mechanics one begins with a space of configurations $\mathcal{C}$ of a system $\mathcal{S}$. Usually the system $\mathcal{S}$ is assumed to be a subsystem of the universe. In this case there is a clock outside the system, which is carried by some inertial observer. This clock is used to label the trajectory of the system in the configuration space $\mathcal{C}$. The classical trajectories are then extrema of some action principle, $\delta I = 0$.

Were it not for the external clock, one could already say that time has disappeared, as each trajectory exists all at once as a curve γ on $\mathcal{C}$. Once the trajectory is chosen, the whole history of the system is determined. In this sense there is nothing in the description that corresponds to what we are used to thinking of as a flow or progression of time. Indeed, just as the whole of any one trajectory exists when any point and velocity are specified, the whole set of trajectories may be said to exist as well, as a timeless set of possibilities.

Time is in fact represented in the description, but it is not in any sense a time that is associated with the system itself. Instead, the t in ordinary classical mechanics refers to a clock carried by an inertial observer, which is not part of the dynamical system being modeled. This external clock is represented in the configuration space description as a special parameterization of each trajectory, according to which the equations of motion are satisfied. Thus, it may be said that there is no sense in which time as something physical is represented in classical mechanics, instead the problem is postponed, as what is represented is time as marked by a clock that exists outside of the physical system which is modeled by the trajectories in the configuration space $\mathcal{C}$.

In quantum mechanics the situation is rather similar. There is a t in the quantum state and the Schroedinger equation, but it is time as measured by an external clock, which is not part of the system being modeled. Thus, when we write,

$$\imath\hbar \frac{d}{dt}\Psi(t) = \hat{H}\Psi(t) \tag{1}$$

the Hamiltonian refers to evolution, as it would be measured by an external observer, who refers to the external clock whose reading is t

The quantum state can be represented as a function Ψ over the configuration space, which is normalizable in some inner product. The inner

[1] For more details and discussion see [1, 2, 3, 4, 5]

product is another a priori structure, it refers also to the external clock, as it is the structure that allows us to represent the conservation of probability as measured by that clock.

When we turn to the problem of constructing a cosmological theory we face a key problem, which is that there is no external clock. There is by definition nothing outside of the system, which means that the interpretation of the theory must be made without reference to anything that is not part of the system which is modeled. In classical cosmological theories, such as general relativity applied to spatially compact universes, or models such as the Bianchi cosmologies or the Barbour-Bertotti model[9, 10], this is expressed by the dynamics having a gauge invariance, which includes arbitrary reparameterizations of the classical trajectories. (In general relativity this is part of the diffeomorphism invariance of the theory.) As a result, the classical theory is expressed in a way that makes no reference to any particular parameterization of the trajectories. Any parameterization is as good as any other, none has any physical meaning. The solutions are then labeled by a trajectory, γ, period, there is no reference to a parameterization.

This is the sense in which time may be said to disappear from classical cosmological theories. There is nothing in the theory that refers to any time at all. At least without a good deal more work, the theory speaks only in terms of the whole history or trajectory, it seems to have nothing to say about what the world is like at a particular moment.

There is one apparently straightforward way out of this, which is to try to define an intrinsic notion of time, in terms of physical observables. One may construct parameter independent observables that describe what is happening at a point on the trajectory if that point can be labeled intrinsically by some physical property. For example, one might consider some particular degree of freedom to be an intrinsic, physical clock, and label the points on the trajectory by its value. This works in some model systems, but in interesting cases such as general relativity it is not known if such an intrinsic notion of time exists which is well defined over the whole of the configuration space.

In the quantum theory there is a corresponding phenomenon. As there is no external t with which to measure evolution of the quantum state one has instead of (1) the quantum constraint equation

$$\hat{H}\Psi = 0 \tag{2}$$

where Ψ is now just a function on the configuration space. Rather than describing evolution, eq. (2) generates arbitrary parameterizations of the trajectories. The wavefunction must be normalizable under an inner product, given by some density ρ on the configuration space. The space of physical states is then given by (2) subject to

$$1 = \int_{\mathcal{C}} \rho \bar{\Psi} \Psi \tag{3}$$

We see that, at least naively time has completely disappeared from the formalism. This has led to what is called the "problem of time in quantum cosmology", which is how to either A) find an interpretation of the theory that restores a role for time or B) provide an interpretation according to which time is not part of a fundamental description of the world, but only reappears in an appropriate classical limit.

There have been various attempts at either direction. We will not describe them here, except to say that, in our opinion, so far none has proved completely satisfactory[2]. There are a number of attempts at A) which succeed when applied to either models or the semiclassical limit, but it is not clear whether any of them overcome technical obstacles of various kinds when applied to the full theory. The most well formulated attempt of type B), which is that of Barbour[5], may very well be logically consistent. But it forces one to swallow quite a radical point of view about the relationship between time and our experience.

Given this situation, we would like to propose that the problem may be not with time, but with the assumptions of the argument that leads to time being absent. Given the number of attempts that have been made to resolve the problem, which have not so far led to a good solution, perhaps it might be better to try to dissolve the issue by questioning one of the assumptions of the argument that leads to the statement of the problem. This is what we would like to do in the following.

3 A problem with the argument for the disappearance of time

Both the classical and quantum mechanical versions of the argument for the disappearance of time begin with the specification of the classical configuration space $\mathcal{C}$. This seems an innocent enough assumption. For a system of N particles in d dimensional Euclidean space, it is simply R^{Nd}. One can then find the corresponding basis of the Hilbert space by simply enumerating the

[2] For good critical reviews that deflate most known proposals, see [1, 2].

Fourier modes. Thus, for cases such as this, it is certainly the case that the configuration space and the Hilbert space structure can be specified *a priori.*

However, there are good reasons to suspect that for cosmological theories it may not be so easy to specify the whole of the configuration or Hilbert space. For example, it is known that the configuration spaces of theories that implement relational notions of space are quite complicated. One example is the Barbour-Bertotti model[9, 10], whose configuration space consists of the relative distances between N particles in d dimensional Euclidean space. While it is presumably specifiable in closed form, this configuration space is rather complicated, as it is the quotient of R^{Nd} by the Euclidean group in d dimensions[5].

The configuration space of compact three geometries is even more complicated, as it is the quotient of the space of metrics by the diffeomorphism group. It is known not to be a manifold everywhere. Furthermore, it has a preferred end, where the volume of the universe vanishes.

These examples serve to show that the configuration spaces of cosmological theories are not simple spaces like R^{Nd}, but may be considerably complicated. This raises a question: could there be a theory so complicated that its space of configurations is not constructible through any finite procedure? For example, is it possible that the topology of an infinite dimensional configuration space were not finitely specifiable? And were this the case, what would be the implications for how we understand dynamics[3]?

We do not know whether in fact the configuration space of general relativity is finitely specifiable. The problem is hard because the physical configuration space is not the space of three metrics. It is instead the space of equivalence classes of three metrics (or connections, in some formalisms) under diffeomorphisms. The problem is that it is not known if there is any effective procedure which will label the equivalence classes.

One can in fact see this issue in one approach to describing the configuration space, due to Newman and Rovelli[12]. There the physical config-

[3] There is an analogous issue in theoretical biology. The problem is that it does not appear that a pre-specifiable set of "functionalities" exists in biology, where pre-specifiable means a compact description of an effective procedure to characterize ahead of time, each member of the set[11, 7]. This problem seems to limit the possibilities of a formal framework for biology in which there is a pre-specified space of states which describe the functionalities of elements of a biological system. Similarly, one may question whether it is in principle possible in economic theory to give in advance an *a priori* list of all the possible kinds of jobs, or goods or services[11].

uration space consists of the diffeomorphism equivalence classes of a set of three flows on a three manifold. (These come from the intersections of the level surfaces of three functions.) These classes are partially characterized by the topologies of the flow lines of the vector fields. We may note that these flow lines may knot and link, thus a part of the problem of specifying the configuration space involves classifying the knotting and linking among the flow lines.

Thus, the configuration space of general relativity cannot be completely described unless the possible ways that flow lines may knot and link in three dimensions are finitely specifiable. It may be noted that there is a decision procedure, due to Hacken, for knots, although it is very cumbersome[13]. However, it is not obvious that this is sufficient to give a decision procedure for configurations in general relativity, because there we are concerned with smooth data. In the smooth category the flow lines may knot and link an infinite number of times in any bounded region. The resulting knots may not be classifiable. All that is known is that knots with a finite number of crossings are classifiable. If these is no decision procedure to classify the knotting and linking of smooth flow lines then the points of the configuration space of general relativity may not be distinguished by any decision procedure. This means that the configuration space is not constructible by any finite procedure.

When we turn from the classical to the quantum theory the same issue arises. First of all, if the configuration space is not constructible through any finite procedure, then there is no finite procedure to define normalizable wave functions on that space. One might still wonder whether there is some constructible basis for the theory. Given the progress of the last few years in quantum gravity we can investigate this question directly, as we know more about the space of quantum states of general relativity than we do about the configuration space of the theory. This is because it has been shown that the space of spatially diffeomorphism invariant states of the quantum gravitational field has a basis which is in one to one correspondence with the diffeomorphism classes of a certain set of embedded, labeled graphs Γ, in a given three manifold Σ [14, 15]. These are arbitrary graphs, whose edges are labeled by spins and whose vertices are labeled by the distinct ways to combine the spins in the edges that meet there quantum mechanically. These graphs are called *spin networks*, they were invented originally by Roger Penrose[16], and then discovered to play this role in quantum gravity[4].

[4]For a review of these developments see [17]. These results have also more recently been

Thus, we cannot label all the basis elements of quantum general relativity unless the diffeomorphism classes of the embeddings of spin networks in a three manifold Σ may be classified. But it is not known whether this is the case. The same procedure that classifies the knots is not, at least as far as is known, extendible to the case of embeddings of graphs.

What if it is the case that the diffeomorphism classes of the embeddings of spin networks cannot be classified? While it may be possible to give a finite procedure that generates all the embeddings of spin networks, if they are not classifiable there will be no finite procedure to tell if a given one produced is or is not the same as a previous network in the list. In this case there will be no finite procedure to write the completeness relation or expand a given state in terms of the basis. There will consequently be no finite procedure to test whether an operator is unitary or not. Without being able to do any of these things, we cannot really say that we have a conventional quantum mechanical description. If spin networks are not classifiable, then we cannot construct the Hilbert space of quantum general relativity.

In this case then the whole set up of the problem of time fails. If the Hilbert space of spatially diffeomorphism invariant states is not constructible, then we cannot formulate a quantum theory of cosmology in these terms. There may be something that corresponds to a "wavefunction of the universe" but it cannot be a vector in a constructible Hilbert space. Similarly, if the configuration space $\mathcal{C}$ of the theory is not constructible, then we cannot describe the quantum state of the universe in terms of a normalizable function on $\mathcal{C}$.

We may note that a similar argument arises for the path integral formulations of quantum gravity. It is definitely known that four manifolds are not classifiable; this means that path integral formulations of quantum gravity that include sums over topologies are not constructible through a finite procedure[20].

Someone may object that these arguments have to do with quantum general relativity, which is in any case unlikely to exist. One might even like to use this problem as an argument against quantum general relativity. However, the argument only uses the kinematics of the theory, which is that the configuration space includes diffeomorphism and gauge invariant classes of some metric or connection. It uses nothing about the actual dynamics

formulated as theorems in a rigorous formulation of diffeomorphism invariant quantum field theories[19, 18].

of the theory, nor does it assume anything about which matter fields are included. Thus, the argument applies to a large class of theories, including supergravity.

4 Can we do physics without a constructible state space?

What if it is the case that the Hilbert space of quantum gravity is not constructible because embedded graphs in three space are not classifiable? How do we do physics? We would like to argue now that there is a straightforward answer to this question. But it is one that necessarily involves the introduction of notions of time and causality.

One model for how to do physics in the absence of a constructible Hilbert space is seen in a recent formulation of the path integral for quantum gravity in terms of spin networks by Markopoulou and Smolin[8][5]. In this case one may begin with an initial spin network Γ_0 with a finite number of edges and nodes (This corresponds to the volume of space being finite.) One then has a finite procedure that constructs a finite set of possible successor spin networks Γ_1^α, where α labels the different possibilities. To each of these the theory associates a quantum amplitude $\mathcal{A}_{\Gamma_0 \rightarrow \Gamma_1^\alpha}$.

The procedure may then be applied to each of these, producing a new set $\Gamma_2^{\alpha\beta}$. Here $\Gamma_2^{\alpha\beta}$ labels the possible successors to each of the Γ_1^α. The procedure may be iterated any finite number of times N, producing a set of spin networks $\mathcal{S}_{\Gamma_0}^N$ that grow out of the initial spin network Γ_0 after N steps. $\mathcal{S}_{\Gamma_0}^N$ is itself a directed graph, where two spin networks are joined if one is a successor of the other. There may be more than one path in $\mathcal{S}_{\Gamma_0}^N$ between Γ_0 and some spin network Γ_{final}. The amplitude for Γ_0 to evolve to Γ_{final} is then the sum over the paths that join them in $\mathcal{S}_{\Gamma_0}^N$, in the limit $N \rightarrow \infty$ of the products of the amplitudes for each step along the way.

For any finite N, $\mathcal{S}_{\Gamma_0}^N$ has a finite number of elements and the procedure is finitely specifiable. There may be issues about taking the limit $N \rightarrow \infty$, but there is no reason to think that they are worse than similar problems in quantum mechanics or quantum field theory. In any case, there is a sense

[5]This followed the development of a Euclidean path integral by Reisenberger[21] and by Reisenberger and Rovelli[22]. Very interesting related work has also been done by John Baez[23]. We may note that the theory described in [8] involves non-embedded spin networks, which probably are classifiable, but it can be extended to give a theory of the evolution of embedded spin networks.

in which each step takes a certain amount of time, in the limit $N \to \infty$ we will be picking up the probability amplitude for the transition to happen in infinite time.

Each step represents a finite time evolution because it corresponds to certain causal processes by which information is propagated in the spin network. The rule by which the amplitude is specified satisfies a principle of causality, by which information about an element of a successor network only depends on a small region of the its predecessor. There are then discrete analogues of light cones and causal structures in the theory. Because the geometry associated to the spin networks is discrete[14], the process by which information at two nearby nodes or edges may propagate to jointly influence the successor network is finite, not infinitesimal.

In ordinary quantum systems it is usually the case that there is a non-vanishing probability for a state to evolve to an infinite number of elements of a basis after a finite amount of time. The procedure we've just described then differs from ordinary quantum mechanics, in that there are a finite number of possible successors for each basis state after a finite evolution. The reason is again causality and discreteness: since the spin networks represent discrete quantum geometries, and since information must only flow to neighboring sites of the graph in a finite series of steps, at each elementary step there are only a finite number of things that can happen.

We may note that if the Hilbert space is not constructible, we cannot ask if this procedure is unitary. But we can still normalize the amplitudes so that the sum of the absolute squares of the amplitudes to evolve from any spin network to its successors is unity. This gives us something weaker than unitarity, but strong enough to guarantee that probability is conserved locally in the space of configurations.

To summarize, in such an approach, the amplitude to evolve from the initial spin network Γ_0 to any element of $\mathcal{S}^N_{\Gamma_0}$, for large finite N is computable, even if it is the case that the spin networks cannot be classified so that the basis itself is not finitely specifiable. Thus, such a procedure gives a way to do quantum physics even for cases in which the Hilbert space is not constructible.

We may make two comments about this form of resolution of the problem. First, it necessarily involves an element of time and causality. The way in which the amplitudes are constructed in the absence of a specifiable basis or Hilbert structure requires a notion of successor states. The theory never has to ask about the whole space of states, it only explores a finite set of successor states at each step. Thus, a notion of time is necessarily introduced.

Second, we might ask how we might formalize such a theory. The role of the space of all states is replaced by the notion of the successor states of a given network. The immediate successors to a graph Γ_0 may be called the *adjacent possible*[7]. They are finite in number and constructible. They replace the idealization of all possible states that is used in ordinary quantum mechanics. We may note a similar notion of an *adjacent possible* set of configurations, reachable from a given configuration in one step, plays a role in formalizations of the self-organization of biological and other complex systems[7].

In such a formulation there is no need to construct the state space *a priori*, or equip it with a structure such as an inner product. One has simply a set of rules by which a set of possible configurations and histories of the universe is constructed by a finite procedure, given any initial state. In a sense it may be said that the system is constructing the space of its possible states and histories as it evolves.

Of course, were we to do this for all initial states, we would have constructed the entire state space of the theory. But there are an infinite number of possible initial states and, as we have been arguing, they may not be classifiable. In this case it is the evolution itself that constructs the subspace of the space of states that is needed to describe the possible futures of any given state. And by doing so the construction gives us an intrinsic notion of time.

5 Conclusions

We must emphasize first of all that these comments are meant to be preliminary. Their ultimate relevance rests partly on the issue of whether there is a decision procedure for spin networks (or perhaps for some extension of them that turns out to be relevant for real quantum gravity[17].). But more importantly, it suggests an alternative type of framework for constructing quantum theories of cosmology, in which there is no *a priori* configuration space or Hilbert space structure, but in which the theory is defined entirely in terms of the sets of adjacent possible configurations, accessible from any given configuration. Whether such formulations turn out to be successful at resolving all the problems of quantum gravity and cosmology is a question that must be left for the future[6].

There are further implications for theories of cosmology, if it turns out to be the case that their configuration space or state space is not finitely constructible. One is to the problem of whether the second law of thermodynamics applies at a cosmological scale. If the configuration space or state space is not constructible, then it is not clear that the ergodic hypothesis is well defined or useful. Neither may the standard formulations of statistical mechanics be applied. What is then needed is a new approach to statistical physics based only on the evolving set of possibilities generated by the evolution from a given initial state. It is possible to speculate whether there may in such a context be a "fourth law" of thermodynamics in which the evolution extremizes the dimension of the adjacent possible, which is the set of states accessible to the system at any stage in its evolution[7].

Finally, we may note that there are other reasons to suppose that a quantum cosmological theory must incorporate some mechanisms analogous to the self-organization of complex systems[6]. For example, these may be necessary to tune the system to the critical behavior necessary for the existence of the classical limit[25, 8]. This may also be necessary if the universe is to have sufficient complexity that a four manifolds worth of spacetime events are completely distinguished by purely relational observables[4, 6]. The arguments given here are complementary to those, and provide yet another way in which notions of self-organization may play a role in a fundamental cosmological theory.

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[6] We may note that the notion of an evolving Hilbert space structure may be considered apart from the issues discussed here[24].

References

[1] C. J. Isham, Canonical Quantum Gravity and the Problem of Time, Imperial College preprint, gr-qc/9210011, /TP/91-92/25.

[2] K. V. Kuchar, *Time and interpretations of quantum gravity* in the *Proceedings of the 4th Canadian Conference on General Relativity and Relativistic Astrophysics*, eds. G. Kunstatter, D. Vincent and J. Williams (World Scientific, Singapore,1992).

[3] C. Rovelli, Phys. Rev. D 42 (1991) 2638; 43 (1991) 442; in *Conceptual Problems of Quantum Gravity* ed. A. Ashtekar and J. Stachel, (Birkhauser,Boston,1991); Class. Quantum Grav. 8 (1991) 317-331.

[4] L. Smolin *Space and time in the quantum universe* in the proceedings of the Osgood Hill conference on *Conceptual Problems in Quantum Gravity* ed. A. Ashtekar and J. Stachel, (Birkhauser,Boston,1991); *Time, structure and evolution in cosmology*, preprint, submitted to the British Journal for the Philosophy of Science; *Time, measurement and information loss in quantum cosmology* gr-qc/9301016 in the Brill Feschrift, ed. Hu and Jacobson, Cambridge University Press, 1994.

[5] J. B. Barbour,to appear in the *Procedings of the NATO Meeting on the Physical Origins of Time Asymmetry* eds. J. J. Halliwell, J. Perez-Mercader and W. H. Zurek (CAmbridge University Press, Cambridge, 1992); *On the origin of structure in the universe* in *Proc. of the Third Workshop on Physical and Philosophical Aspects of our Understanding of Space and Time* ed. I. O. Stamatescu (Klett Cotta); *Time and the interpretation of quantum gravity* Syracuse University Preprint, April 1992.

[6] L. Smolin, *The Life of the Cosmos* Oxford University Press, New York and Weidenfeld and Nicolson, London, (1997).

[7] S. Kauffman, *Investigations on the character of autonomous agents and the worlds they mutually create* Santa Fe Institute preprint, 1996.

[8] F. Markopoulou and L. Smolin, *Causal evolution of spin networks* gr-qc/9702025. CGPG 97/2-1.

[9] J. B. Barbour, Nature 249 (1974) 328 Erattum: Nature 250 (1974) 606; Nuovo Cimento 26B (1975) 16; J.B. Barbour and B. Bertotti, Nouvo Cimento 38B (1977) 1.

[10] J.B. Barbour and B. Bertotti, Proc. Roy. Soc. Lond. A 382 (1982) 295.

[11] S. Kauffman, *Origins of Order* Oxford University Press; *At home in the universe*, Oxford University Press, 1996.

[12] E. T. Newman and C. Rovelli, Physical Review Letters, (1992)

[13] J. Hemion *Classification of knots in three dimensional space* Oxford University Press.

[14] C. Rovelli and L. Smolin *Discreteness of area and volume in quantum gravity* Nuclear Physics B 442 (1995) 593. Erratum: Nucl. Phys. B 456 (1995) 734.

[15] C. Rovelli and L. Smolin, "Spin networks and quantum gravity" gr-qc/9505006, Physical Review D 52 (1995) 5743-5759.

[16] R Penrose: in *Quantum theory and beyond* ed T Bastin, Cambridge U Press 1971; in *Advances in Twistor Theory*, ed. L. P. Hughston and R. S. Ward, (Pitman,1979) p. 301; in *Combinatorial Mathematics and its Application* (ed. D. J. A. Welsh) (Academic Press,1971).

[17] L. Smolin, *The future of spin networks* gr-qc/9702030.

[18] J. Baez, Spin networks in gauge theory, Adv. Math. **117** (1996), 253-272, gr-qc/941107; Spin networks in nonperturbative quantum gravity, in *The Interface of Knots and Physics*, ed. Louis Kauffman, American Mathematical Society, Providence, Rhode Island, 1996, gr-qc/9504036.

[19] A. Ashtekar and C. J. Isham, Class and Quant Grav 9 (1992) 1069; A Ashtekar J Lewandowski D Marlof J Mourãu T Thiemann: "Quantization of diffeomorphism invariant theories of connections with local degrees of freedom", gr-qc/9504018, JMP 36 (1995) 519; T. Thiemann, Quantum spin dynamics I, Harvard preprint (1996), gr-qc/9606089; Quantum spin dynamics II, Harvard preprint (1996), gr-qc/9606090; A. Ashtekar and J. Lewandowski, "Quantum Geometry I: area operator" gr-qc/9602046.

[20] J.B. Hartle, *Unruly Topologies in Two Dimensional Quantum Gravity*, Class. & Quant. Grav., 2, 707, 1985b; R. Geroch and J.B. Hartle, *Computability and Physical Theories* Found. Phys., 16, 533, 1986. J.B. Hartle, *Unruly Topologies in Two Dimensional Quantum Gravity*, Class. & Quant. Grav., 2, 707, 1985b.

[21] M. Reisenberger, unpublished.

[22] M. Reisenberger and C. Rovelli, *"Sum over Surfaces" form of Loop Quantum Gravity*, gr-qc/9612035.

[23] J. Baez, seminars, Penn State, February 1997.

[24] R. Doldan, R. Gambini and P. Mora, Int. J. of Theor. Phys. 35 (1996) 2057.

[25] L. Smolin, *Cosmology as a problem in critical phenomena* in the proceedings of the Guanajuato Conference on Complex systems and binary networks, (Springer,1995), eds. R. Lopez-Pena, R. Capovilla, R. Garcia-Pelayo, H. Waalebroeck and F. Zertuche. gr-qc/9505022.