

# Prima Facie Questions in Quantum Gravity<sup>1</sup>

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## Abstract

The long history of the study of quantum gravity has thrown up a complex web of ideas and approaches. The aim of this article is to unravel this web a little by analysing some of the *prima facie* questions that can be asked of almost any approach to quantum gravity and whose answers assist in classifying the different schemes. Particular emphasis is placed on (i) the role of background conceptual and technical structure; (ii) the role of spacetime diffeomorphisms; and (iii) the problem of time.

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# 1 Introduction

## 1.1 Preliminary Remarks

The many ideas and suggestions that have become attached to the study of quantum gravity form a substantial web whose subtle interconnections often cause considerable confusion, particularly amongst those approaching the subject for the first time. Therefore, when seeking to assess any particular scheme (such as canonical quantisation), it is helpful to begin by looking at the subject in the broadest possible terms with the aim of unravelling this web a little.

The present essay is intended to serve this need via an examination of certain *prima facie* questions that can be used to clarify the different structural and conceptual frameworks adopted by the various approaches to quantum gravity. The paper starts with a brief discussion of the four general ways whereby a quantum theory of gravity might be constructed. This is followed by some motivation for studying the subject in the first place and an explanation of what is meant by a ‘*prima facie*’ question. Next there is a short sketch of the major current research programmes in quantum gravity: this ensures that the subsequent discussion does not take place in a complete technical vacuum. Then we move to three questions that are of exceptional importance in quantum gravity: (i) the role of background structure; (ii) the role of the spacetime diffeomorphism group; and (iii) the ‘problem of time’. Finally, several of the current approaches to quantum gravity are used to illustrate some of the different ways in which these fundamental issues can be addressed.

What follows is a pedagogical exposition of some basic ideas in quantum gravity. It is *not* a full review of the field and, for this reason, references to original work are limited in number. Comprehensive reference lists can be found in recent genuine reviews of quantum gravity: for example [Alv89], [Kuc92] and [Ish92, Ish93].

## 1.2 What is Quantum Gravity?

Research in quantum gravity could perhaps be defined as any attempt to construct a theoretical scheme in which ideas from general relativity and quantum theory appear together in some way. A fundamental property of any such scheme is the existence of units with *dimensions* formed from Newton’s constant  $G$ , Planck’s constant  $\hbar$ , and the ubiquitous speed of light  $c$ . For example

- the Planck length  $L_P := (G\hbar/c^3)^{1/2} \simeq 10^{-33}cm$ ;
- the Planck time  $T_P := L_P/c \simeq 10^{-42}s$ ;
- the Planck mass  $M_P := \hbar/cL_P \simeq 10^{-5}gms$ ;
- the Planck energy  $E_P = M_Pc^2 \simeq 10^{18}GeV$ ;

A major question in any approach to quantum gravity is what role is played by these fundamental units.

This definition of quantum gravity is very broad and includes, for example, studies of a quantum field propagating in a spacetime manifold equipped with a fixed background Lorentzian metric. However, in practice, references to ‘quantum gravity’ usually include the idea that a quantum interaction of the gravitational field with itself is involved in some way; quantum field theory in a fixed background is then better regarded as a way of probing certain aspects of quantum gravity proper.

Understood in this more limited sense, attempts to construct a quantum theory of gravity can be divided into four broad categories that I shall refer to as type I, type II, type III, and type IV.

#### I. The *quantisation* of general relativity.

The idea is to start with the classical theory of general relativity and then to apply some type of quantisation algorithm. This is intended to be analogous to the way in which the classical theory of an atom bound by the Coulomb potential is ‘quantised’ by replacing certain classical observables with self-adjoint operators on a Hilbert space. Of course, this is essentially also the approach used in developing important elementary-particle physics ideas like the Salam-Weinberg electro-weak theory and the quantum chromodynamics description of the strong nuclear force.

Approaches to quantum gravity of this type have been studied extensively and divide into two main categories: (i) ‘canonical’ schemes that start with a pre-quantum division of four-dimensional spacetime into a three-dimensional space plus time; and (ii) ‘covariant’ schemes that try to apply quantum ideas in a full spacetime context. The perturbative non-renormalisability of covariant quantum gravity was proved in the early 1970s, and most activity in this area ceased thereafter. On the other hand, the major advance inaugurated by Ashtekar’s work has led to the canonical approach becoming one of the most active branches of quantum gravity research [Ash91].

#### II. *General-relativise* quantum theory.

Schemes of this type are much rarer than those of type I. The main idea is to begin with some prior idea of quantum theory and then to force it to be compatible with general relativity. The biggest programme of this type is due to Haag and his collaborators [FH87].

III. General relativity appears as a *low-energy limit* of a theory that is formed using conventional quantum ideas but which does *not* involve a type-I quantisation of the classical theory of relativity.

The dimensional nature of the basic Planck units lends credence to the idea of a theory that could reproduce standard general relativity in regimes whose scales are well away from that of the Planck time, length, energy *etc.* Superstring theory is the most successful scheme of this type and has been the subject of intense study during the last decade.

**IV.** Both general relativity *and* standard quantum theory appear only in certain limiting situations in the context of a theory that starts from radically new perspectives.

Very little is known about potential schemes of this type or, indeed, if it is necessary to adopt such an iconoclastic position in order to solve the problem of quantum gravity. However, the recurring interest in such a possibility is based on the frequently espoused view that the basic ideas behind general relativity and quantum theory are fundamentally incompatible and that any complete reconciliation will necessitate a total rethinking of the central categories of space, time and matter.

### 1.3 Why Should We Study Quantum Gravity?

Notwithstanding many decades of intense work, we are still far from having a complete quantum theory of gravity. The problem is compounded by the total lack of any empirical data (either observational or experimental) that is manifestly relevant to the problem. Under these circumstances, some motivation is necessary to explain why we should bother with the subject at all.

- *We must say something.* The value of the Planck length suggests that quantum gravity should be quite irrelevant to, for example, atomic physics. However, the non-renormalisability of the perturbative theory means it is impossible to actually compute these corrections, even if physical intuition suggests they will be minute. Furthermore, no consistent theory is known in which the gravitational field is left completely classical. Hence we are obliged to say *something* about quantum gravity, even if the final results will be negligible in all normal physical domains.
- *Gravitational singularities.* The classical theory of general relativity is notorious for the existence of unavoidable spacetime singularities. It has long been suggested that a quantum theory of gravity might cure this disease by some sort of ‘quantum smearing’.
- *Quantum cosmology.* A particularly interesting singularity is that at the beginning of a cosmological model described by, say, a Robertson-Walker metric. Classical physics breaks down here, but one of the aims of quantum gravity has always been to describe the ‘origin’ of the universe as some type of quantum event.
- *The end state of the Hawking radiation process.* One of the most striking results involving general relativity and quantum theory is undoubtedly Hawking’s famous discovery of the quantum thermal radiation produced by a black hole. Very little is known of the final fate of such a system, and this is often taken to be another task for a quantum theory of gravity.
- *The unification of fundamental forces.* The weak and electromagnetic forces are neatly unified in the Salam-Weinberg model, and there has also been a partial unification with the strong force. It is an attractive idea that a consistent quantum theory of gravity *must* include a unification of all the fundamental forces.

- *The possibility of a radical change in basic physics.* The deep incompatibilities between the basic structures of general relativity and of quantum theory have lead many people to feel that the construction of a consistent theory of quantum gravity requires a profound revision of the most fundamental ideas of modern physics. The hope of securing such a paradigm shift has always been a major reason for studying the subject.

## 1.4 What Are Prima Facie Questions?

By a ‘prima facie’ question I mean the type of question that can be asked of almost any approach to quantum gravity and which is concerned with the most basic issues in the subject. The following general classes are of particular importance:

1. *General issues concerning the relation between classical and quantum physics.* The minimal requirement of any quantum theory of gravity is that it should reproduce classical general relativity and standard quantum theory in the appropriate physical domains. However, it is difficult to make general categorical statements about either (i) what is meant by the ‘classical limit’ of a given quantum system; or (ii) how to construct a quantum analogue of any given classical system: in practice, the matter is usually decided on an *ad hominum* basis. This produces significant problems in attempts to quantise gravity using, for example, a type-I canonical formalism.

2. *Specific issues in quantum gravity.* Three especially important issues of direct relevance to quantum gravity are as follows:

- How relevant are the *spacetime* concepts associated with the classical theory of general relativity? Do the Planck length and time signify the scales at which all normal ideas of space and time break down?
- To what extent is it appropriate to construct a quantum theory of gravity using the technical and conceptual apparatus of *standard* quantum theory? For example, the traditional ‘Copenhagen’ interpretation of quantum theory is often asserted to be quite inappropriate for a theory of quantum cosmology. What should take its place?
- Does a consistent quantum theory of gravity necessarily involve the *unification* of the fundamental forces of physics or is it possible to construct a theory that involves the gravitational field alone? This questions signals one of the key differences between superstring theory and the Ashtekar version of canonical quantum gravity: the latter asserts that a quantisation of pure gravity *is* possible whereas one of the main claims of superstring theory is to provide a scheme that encompasses all the forces.

Most readers will probably agree that questions like the above are likely to be relevant in any approach to constructing a quantum theory of gravity. But there is a hidden danger that should not be underestimated. As theoretical physicists, we are inclined

towards a simple realist philosophy that sees our professional activities in terms of using an appropriate conceptual scheme to link what is ‘out there’ (the world of ‘actual facts’) to some mathematical model. However, one of the important lessons from the philosophy of science is that facts and theories cannot be so neatly separated: what we call a ‘fact’ does not exist without some theoretical schema for organising experimental and experiential data; and, conversely, in constructing a theory we inevitably impose some prior idea of what we mean by a fact.

In most branches of physics no real problem arises when handling this interconnected triad of facts, mathematical model and bridging conceptual framework: it has simply become part of the standard methodology of science. However, the situation in quantum gravity is rather different since there are no known ways of directly probing the Planck regime. This lack of hard empirical data means that research in the subject has tended to focus on the construction of abstract theoretical schemes that are (i) internally consistent (in a mathematical sense), and (ii) are compatible with some preconceived set of concepts. This rather introspective situation helps to fuel the recurrent debate about whether the construction of a comprehensive theory of quantum gravity requires a preliminary fundamental reappraisal of our standard concepts of space, time and matter, or whether it is better to try first to construct an internally consistent mathematical model and only then to worry about what it ‘means’.

Of course, a sensible pragmatist will strive to maintain a proper balance between these two positions, but this ever-present tension between conceptual framework and mathematical model does lend a peculiar flavour to much research in the field. In particular, the wide range of views on how to approach the subject has generated a variety of different research programmes whose practitioners not infrequently have difficulty in understanding what members of rival schools are trying to do. This is one reason why it is important to uncover as many as possible of the (possibly hidden) assumptions that lie behind each approach: one person’s ‘deep’ problem may seem irrelevant to another simply because the starting positions are so different. This situation also shows how important it is to try to find some area of physics where the theory can be tested directly. A particularly important question in this context is whether there are genuine quantum gravity effects at scales well below the Planck energy. Needless to say, the answer to a question like this is itself likely to be strongly theory dependent.

## 1.5 Current Research Programmes in Quantum Gravity

At this stage it might be helpful to give a brief account of some of the major current research programmes in quantum gravity. I find the following scheme particularly useful, although the plethora of topics studied could certainly be organised in many other ways.

### A. *Quantum Gravity Proper*

The two major current programmes that attempt to construct a full-blown theory of quantum gravity are the Ashtekar version of canonical quantum gravity, and superstring theory.

Canonical quantum gravity has a long history and has been used extensively to discuss a variety of conceptual issues including the problem of time and the possible meaning of a quantum state of the entire universe. However, the great difficulties that arise when trying to make proper mathematical sense of the crucial equations (in particular, the Wheeler-DeWitt equation) that arise in this type-I approach eventually impose insurmountable obstacles. One of the reasons why the Ashtekar programme is so interesting is that many of these issues can now be reopened within the context of a mathematical framework that is much better behaved.

The technical and conceptual framework within which superstring theory is currently discussed is very different from that of canonical quantum gravity and owes far more to its origins in elementary particle physics than it does to the classical theory of general relativity, which arises only as a low-energy limit of the theory. As a consequence, questions concerning the status of space and time take on a quite different form from that in the canonical formalism. This is to be expected of any type-III approach to quantum gravity.

Another significant type-I scheme is the ‘euclidean programme’ whose basic ingredient is a functional integral over metrics with a Riemannian signature [GH93]. This view of quantum gravity cannot be called a ‘full-blown’ approach to quantum gravity since, as yet, there is no known way of making mathematical sense of such integrals; in practice, most work involves a saddle-point, semi-classical approximation. On the other hand, the heuristic functional integral can easily be extended to include a sum over different manifolds, and hence the scheme is a natural one in which to discuss topology change; something that is rather difficult in the canonical approach.

### B. *Quantum Cosmology*

One of the major reasons for studying quantum gravity is to understand the Planck era of the very early universe. Most discussions of quantum cosmology have employed ‘minisuperspace’ models (sometimes in the context of the euclidean programme) in which only a finite number of the gravitational modes are quantised. Too much weight should not be attached to the results of such crude approximations, especially those (the great bulk) that use the ill-defined equations of standard canonical quantum gravity. However, models of this type can be valuable tools for exploring the many conceptual problems that arise in the typical quantum-cosmology situation where one aspires to describe the quantum state of the entire universe. For example, there has been much discussion of the problem of time and the, not unrelated, inapplicability of the normal Copenhagen interpretation of quantum theory. One of the currently most active ways of tackling these issues involves the ‘consistent histories’ approach to quantum theory [Har93].

### C. *Model Systems*

In addition to minisuperspace techniques many other model systems have been studied with the aim of probing specific aspects of the full theory. For example:

- *Quantum gravity theories in three or less spacetime dimensions.* Low-dimensional quantum gravity has been widely studied. In particular, the three-dimensional

theory has been solved completely using a variety of different methods and provides valuable information on how these might be related in general.

- *Quantum field theory on a spacetime with a fixed Lorentzian geometry.* There has been a substantial interest in this subject since Hawking’s discovery of black-hole radiation. It throws useful light on certain aspects of the full theory and might also have direct astrophysical significance.
- *Semi-classical quantisation.* There has been much activity in recent years devoted to the WKB approximation to the Wheeler-DeWitt equation. This approach is unlikely to reveal much about the Planck regime proper but, nevertheless, there have been a number of interesting results, especially suggestions that the techniques may yield genuine results away from the scales set by the Planck length and time.
- *Regge calculus.* The idea of approximating spacetime by a simplicial complex has been of interest in both classical and quantum gravity for a long time. The quantum results to date are rather modest but could increase in the future with the rapid increase in the power of computing systems.

#### D. *Spacetime Structure at the Planck Length and Time*

A major motivation for many who elect to study quantum gravity is the belief that something really fundamental happens to the structure of space and time at the Planck scale. This has inspired a number of fragmented attempts that start *ab initio* with a theoretical framework in which standard spacetime concepts are radically altered. Schemes of this type include twistor theory, various approaches to a discrete models of physics, non-commutative geometry, quantum topology or set theory, quantum causal sets, and the like. The main difficulty is that the starting point of programmes of this type is so far from conventional physics that it is difficult to get back to the mundane world of Einstein field equations in a continuum spacetime.

## 2 Prima Facie Questions in Quantum Gravity

I wish now to consider in more detail three exceptionally important questions that can be asked of any approach to quantum gravity. These are: (i) what background structure is assumed?; (ii) what role is played by the spacetime diffeomorphism group?; (iii) how is the concept of ‘time’ to be understood?

### 2.1 Background Structure

The phrase ‘background structure’ can mean several things. It can refer to a specific choice of, say, a manifold or Lorentzian metric that is fixed once and for all and which therefore is not itself subject to quantum effects. For example, the early attempts to construct a quantum theory of gravity using ideas drawn from particle physics involved

writing the spacetime metric  $g_{\alpha\beta}(X)$  as a sum  $\eta_{\alpha\beta} + h_{\alpha\beta}(X)$  where  $\eta_{\alpha\beta}$  is the Minkowski metric  $\text{diag}(-1, 1, 1, 1)$ . The idea was to quantise  $h_{\alpha\beta}(X)$  using standard relativistic quantum field theory (see section 3.1). Thus, in this type-I approach, the topology and differential structure of spacetime are fixed as that of the vector space  $\mathbb{R}^4$ , while the background Minkowski metric plays a key role in the quantum field theory via its associated Poincaré group of isometries.

Analysis of such background structure is a useful aid in classifying and distinguishing the various approaches to quantum gravity. However, ‘background’ can be used in another way that, if anything, is even more important but which is frequently not articulated in such a concrete way. I mean the entire *conceptual* and *structural* framework within whose language any particular approach is couched. Different approaches to quantum gravity differ significantly in the frameworks they adopt, which causes no harm—indeed the selection of such a framework is an essential pre-requisite for theoretical research—provided the choice is made consciously. The problems arise when practitioners from one particular school become so accustomed to a specific structure that it becomes, for them, an almost *a priori* set of truths, and then they find it impossible to understand how any other position could ever be valid. Unfortunately, a fair number of such misunderstandings have occurred during the history of the study of quantum gravity.

Bearing all this in mind let us begin now to examine some of the specific issues that concern the choice of such technical or conceptual background structure.

#### A. The Use of Standard Quantum Theory

One of the common features of superstring theory and the Ashtekar programme is their use of standard quantum theory. True, the normal formalism has to be adapted to handle the constraints that appear in both approaches, but most of the familiar apparatus is present: linear vector space, linear operators, inner products *etc.* However, there have been recurrent suggestions that a significant change in the formalism is necessary before embarking on a full quantum gravity programme. For example:

1. General relativity may induce an essential *non-linearity* into quantum theory [KFL86, Pen86, Pen87]. In particular, it may be possible to regard the infamous ‘reduction of the state vector’ as a genuine dynamical process induced by interactions involving the gravitational field. The major approaches to quantum gravity would change radically if it was necessary to start *ab initio* with a non-linear theory rather than, say, deriving the non-linear effects as some type of higher-order correction.
2. How valid are the *continuum* concepts employed in quantum theory, in particular the use of real and complex numbers? The idea here is roughly as follows. One reply to the question “why do we use real numbers in quantum theory?” is that we want the eigenvalues of self-adjoint operators to be real numbers because they represent the possible results of physical measurements. And why should the results of measurements be represented by real numbers? Because all measurements can ultimately be reduced to the positions of a pointer in space, and space is modelled using real numbers. In other words, in using real or complex numbers in

quantum theory we are arguably making a prior assumption about the continuum nature of space. However, it has often been suggested that the Planck length and time signal the scale at which standard spacetime concepts break down, and that a more accurate picture might be a discrete structure that looks like a familiar differentiable manifold only in some coarse-grained sense. But we will not be able to construct such a theory if we start with a quantum framework in which the continuum picture has been assumed *a priori*.

This argument is not water-tight, but it does illustrate quite well how potentially unwarranted assumptions can enter speculative theoretical physics and thereby undermine the enterprise.

3. The Hawking radiation from a black hole is associated with a loss of information through the event horizon and corresponds to what an external observer would regard as a transition from a pure state to a mixed state. If the idea of a potential loss of information has to be imposed on the formalism *ab initio* it would enforce a significant change in the current quantum gravity programmes.

These three possible objections to standard quantum theory are all concerned with the mathematical structure of the subject. However, another serious problem arises with the *interpretation* of quantum theory, especially in the context of quantum cosmology. This particular objection has been taken very seriously in recent years and by now there is a fairly widespread agreement that the familiar ‘Copenhagen’ view is not appropriate. In particular, there is a strong desire to find an alternative interpretation whose fundamental ingredients do not include the notion of a measurement by an external observer.

The consistent-histories approach may provide such a scheme although, even here, there is a problem in so far as most discussions of this subject presuppose the existence of a fixed spacetime. In conventional quantum theory there is certainly a strong case for arguing that this is necessary, both for the mathematical foundations and for the conceptual interpretation of the theory. This raises what turns out to be one of the most interesting *prima facie* questions in quantum gravity: how much of the standard spacetime structure must be imposed as part of the fixed background?

#### B. *How Much Spacetime Structure Must Be Fixed?*

The mathematical model of spacetime used in classical general relativity is a differentiable manifold equipped with a Lorentzian metric. Some of the more important pieces of substructure underlying this picture are illustrated in Figure 1.

The bottom level is a set  $M$  whose elements are to be identified with spacetime ‘points’ or ‘events’. This set is formless with its only general mathematical property being the cardinal number. In particular, there are no relations between the elements of  $M$  and no special way of labelling any such element.

The next step is to impose a topology on  $M$  so that each point acquires a family of neighbourhoods. It now becomes possible to talk about relationships between points, albeit in a rather non-physical way. This defect is overcome by adding the key ingredient

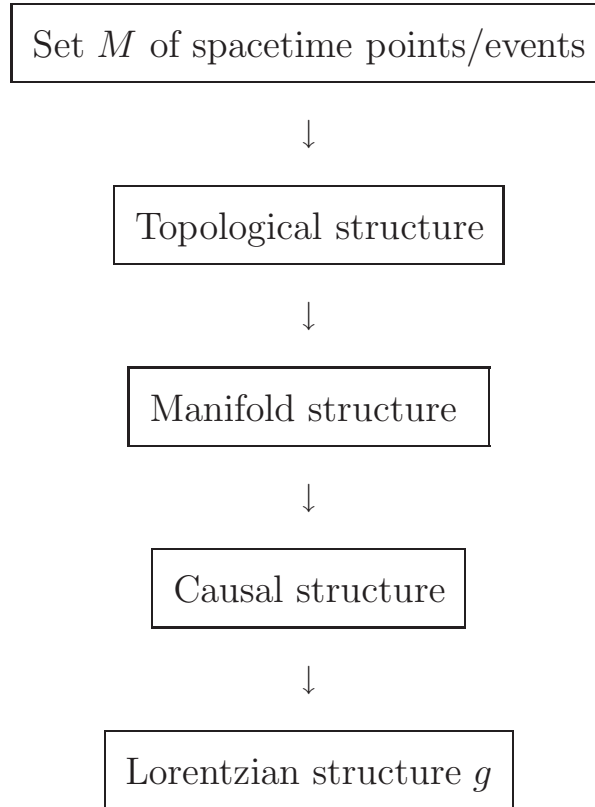


Figure 1. The spacetime structure of classical general relativity

of all standard views of spacetime: the topology of  $M$  must be compatible with that of a differentiable manifold. A point can then be labelled uniquely in  $M$  (at least, locally) by giving the values of four real numbers. Such a coordinate system also provides a more specific way of describing relationships between points of  $M$ , albeit not intrinsically in so far as these depend on which coordinate systems are chosen to cover  $M$ .

In the final step a Lorentzian metric  $g$  is placed on  $M$ , thereby introducing the ideas of the lengths of a path joining two spacetime points, parallel transport with respect to a Riemannian connection, causal relations between pairs of points *etc.* There are also a variety of possible intermediate steps between the manifold and Lorentzian pictures; for example, as signified in Figure 1, the idea of a causal structure is more primitive than that of a Lorentzian metric.

The key question is how much of this classical structure is to be held fixed in the quantum theory. In the context of the Copenhagen interpretation the answer is arguably “all of it”. I suspect that Bohr would have identified the spacetime Lorentzian structure as an intrinsic part of that classical world which he felt was such an essential epistemological prerequisite for the discussion of quantum objects. It seems probable therefore that he would not have approved at all of the subject of quantum gravity!

However, since we wish to assert that some sort of quantum spacetime structure *is* meaningful, the key question for any particular approach to quantum gravity is how *much*

of the hierarchy illustrated in Figure 1 must be kept fixed. For example, in most type-I approaches to ‘quantising’ the gravitational field, the set of spacetime points, topology and differential structure are all fixed, and only the Lorentzian metric  $g$  is subject to quantum fluctuations. But one can envisage more interesting type-I possibilities in which, for example, the set  $M$  and its topology are fixed, but quantum fluctuations are permitted over all those manifold structures that are compatible with this particular topology. Moving back a step, one can envisage the exotic idea of fixing only the set-theoretic structure of  $M$  and allowing quantum fluctuations over topologies that can be placed on this set, including perhaps many that are not compatible with a manifold structure at all. Finally, one might even imagine ‘quantising’ the point set  $M$  itself: presumably by allowing quantum fluctuations in its cardinal number.

The ideas sketched above are all examples of what might be called ‘horizontal’ quantisation in which the quantum fluctuations take place only within the category of objects specified by the classical theory. However, it is also possible to contemplate ‘vertical’ quantisation in which fluctuations take place in a wider category. A simple example, and one that arises in several type-I schemes, is to permit quantum fluctuations in the metric to include fields that are degenerate, or have signature other than  $(-1, 1, 1, 1)$ . A more exotic possibility would be to allow fluctuations in manifold structure that include non-commutative manifolds.

The ideas of ‘horizontal’ and ‘vertical’ quantisation arise most naturally in the context of type-I approaches in which a given classical system is ‘quantised’ in some way. However, the question of how much of the classical structure of spacetime remains at different levels of the full theory can be asked meaningfully in all the four general approaches to quantum gravity. This is related to the question of the role of the Planck length in such theories. A common expectation is that the standard picture of space and time is applicable only at scales well above the Planck regime, and that the Planck length, time, energy *etc* signal the point at which *phase transitions* take place.

The notion of different phases is attractive but it also suggests that a complete theory of quantum gravity should assume no prior spacetime structure at all. Of course, this does not forbid the construction of partial theories that describe the theory in a particular phase; indeed, this may be a necessary first stage in the construction of the full structure.

### C. Background Causal Structure

A piece of potential background of particular importance is causal structure. For example, consider the problem of constructing the quantum theory of a scalar field  $\phi$  propagating on a spacetime manifold  $M$  equipped with a fixed Lorentzian metric  $g$ . In such a theory, a key role is played by the microcausal condition

$$[\hat{\phi}(X), \hat{\phi}(Y)] = 0 \tag{1}$$

for all spacetime points  $X$  and  $Y$  that are spacelike separated with respect to  $g$ . More rigorously, if  $\mathcal{A}(O)$  denotes the  $C^*$ -algebra of local observables associated with the spacetime region  $O \subset M$ , then we require  $[\mathcal{A}(O_1), \mathcal{A}(O_2)] = 0$  for any disjoint regions  $O_1$  and  $O_2$  that are spacelike separated.

Now consider what happens in a type-I, ‘covariant’ approach to quantum gravity. If the Lorentzian metric  $g$  becomes quantised then the light cone associated with any spacetime point is no longer fixed and it is not meaningful to impose a microcausal relation like (1): any pair of spacetime points are ‘potentially’ null or time-like separated, and hence spacetime quantum fields can never commute.

This collapse of one of the bedrocks of conventional quantum field theory is probably the single greatest reason why spacetime approaches to quantum gravity have not got as far as might have been hoped. In the original, particle-physics based schemes the problem was circumscribed by introducing a background Minkowskian metric  $\eta$  and then quantising the graviton using the microcausal structure associated with  $\eta$ . Such a background is also necessary for the idea of ‘short-distance’ behaviour (which plays a key role in discussing renormalisability) to have any meaning. However, the use of a fixed causal structure is an anathema to most general relativists and therefore, even if this approach to quantum gravity had worked (which it did not), there would still have been a strong compunction to reconstruct the theory in a way that does not employ any such background.

This very non-trivial problem is one of the reasons why the canonical approach to quantum gravity has been so popular. However, a causal problem arises here too. For example, in the Wheeler-DeWitt approach, the configuration variable of the system is the Riemannian metric  $q_{ab}(x)$  on a three-manifold  $\Sigma$ , and the canonical commutation relations invariably include the set

$$[\hat{q}_{ab}(x), \hat{q}_{cd}(x')] = 0 \quad (2)$$

for all points  $x$  and  $x'$  in  $\Sigma$ . In normal canonical quantum field theory such a relation arises because  $\Sigma$  is a space-like subset of spacetime, and hence the fields at  $x$  and  $x'$  should be simultaneously measurable. But how can such a relation be justified in a theory that has no fixed causal structure? This problem is rarely mentioned but it means that, in this respect, the canonical approach to quantum gravity is no better than the covariant one. It is another aspect of the ‘problem of time’ to which my second lecture is devoted

## 2.2 The Role of the Spacetime Diffeomorphism Group $\text{Diff}(M)$

The group  $\text{Diff}(M)$  of spacetime diffeomorphisms plays a key role in the classical theory of general relativity and so the question of its status in quantum gravity is of considerable *prima facie* interest. We shall restrict our attention to diffeomorphisms with compact support, by which I mean those that are equal to the unit map outside some closed and bounded region of  $M$ . Thus, for example, a Poincaré-group transformation of Minkowski spacetime is not deemed to belong to  $\text{Diff}(M)$ . This restriction is imposed because the role of transformations with a non-trivial action in the asymptotic regions of  $M$  is quite different from those that act trivially.

The role of  $\text{Diff}(M)$  in quantum gravity depends strongly on the approach taken to the subject. For example, in a type-III or type-IV scheme the structure of classical relativity is expected to appear only in a low-energy limit and so there is no strong

reason to suppose that  $\text{Diff}(M)$  will play any fundamental role in the quantum theory. A type-II scheme is quite different since the group of spacetime diffeomorphisms is likely to be a key ingredient in forcing a quantum theory to comply with the demands of general relativity. On the other hand, the situation for type-I approaches is less clear. Any scheme based on a prior canonical decomposition into space plus time is bound to obscure the role of spacetime diffeomorphisms, and even in the covariant approaches quantisation may affect enough of the classical theory to detract from the significance of such transformations.

Some insight can be gained by looking at certain aspects of spacetime diffeomorphisms in classical general relativity. It is helpful here to distinguish between the pseudo-group of local coordinate transformations and the genuine group  $\text{Diff}(M)$  of global diffeomorphisms of  $M$ . Compatibility with the former can be taken to imply that the theory should be written using tensorial objects on  $M$ . On the other hand, as Einstein often emphasised,  $\text{Diff}(M)$  appears as an active group of transformations of  $M$ , and invariance under this group implies that the points in  $M$  have no direct physical significance. Of course, this is also true in special relativity but it is mitigated there by the existence of inertial reference frames that can be transformed into each other by the Poincaré group of isometries of the Minkowski metric.

Put somewhat differently, the action of  $\text{Diff}(M)$  on  $M$  induces an action on the space  $\mathcal{F}$  of spacetime fields, and the only thing that has immediate physical meaning is the quotient space  $\mathcal{F}/\text{Diff}(M)$  of orbits, *i.e.*, two field configurations are regarded as physically equivalent if they are connected by a  $\text{Diff}(M)$  transformation. Technically, this is analogous to the situation in electromagnetism whereby a vector potential  $A_\mu$  is equivalent to  $A_\mu + \partial_\mu f$  for all functions  $f$ . However, there is an important difference between the electromagnetism and general-relativity. Electromagnetic gauge transformations occur at a fixed spacetime point  $X$ , and the physical configurations can be identified with the values of the electromagnetic field  $F_{\mu\nu}(X)$ , which depends *locally* on points of  $M$ . On the other hand,  $\text{Diff}(M)$  maps one spacetime point into another, and therefore the obvious way of constructing a  $\text{Diff}(M)$ -invariant object is to take some scalar function of spacetime fields and integrate it over the whole of  $M$ , which gives something that is very *non-local*. The idea that ‘physical observables’ are naturally non-local is an important ingredient in many approaches to quantum gravity.

## 2.3 The Problem of Time

One of the major issues in quantum gravity is the so-called ‘problem of time’. This arises from the very different roles played by the concept of time in quantum theory and in general relativity. Let us start by considering standard quantum theory.

1. Time is not a physical observable in the normal sense since it is not represented by an operator. Rather, it is treated as a *background* parameter which, as in classical physics, is used to mark the evolution of the system; in this sense it can be regarded as part of Bohr’s background classical structure. In particular, it

provides the parameter  $t$  in the time-dependent Schrödinger equation

$$i\hbar \frac{d\psi_t}{dt} = \hat{H}\psi_t. \quad (3)$$

This is why the meaning assigned to the time-energy uncertainty relation  $\delta t \delta E \geq \frac{1}{2}\hbar$  is quite different from that associated with, for example, the position and the momentum of a particle.

2. The idea of events happening at a single time plays a crucial role in the technical and conceptual foundations of quantum theory:
  - The notion of a *measurement* made at a particular time is a fundamental ingredient in the conventional Copenhagen interpretation. In particular, an *observable* is something whose value can be measured at a fixed time.
  - One of the central requirements of the *scalar product* on the Hilbert space of states is that it is conserved under the time evolution (3). This is closely connected to the unitarity requirement that probabilities always sum to one.
  - More generally, a key ingredient in the construction of the Hilbert space for a quantum system is the selection of a complete set of observables that are required to *commute* at a fixed value of time.
3. These ideas can be extended to systems that are compatible with special relativity: the unique time system of Newtonian physics is simply replaced with the set of relativistic inertial reference frames. The quantum theory can be made independent of a choice of frame if it carries a unitary representation of the Poincaré group. In the case of a relativistic quantum field theory, this is closely related to the micro-causality requirement, which—as emphasised earlier—becomes meaningless if the light cone is itself the subject of quantum fluctuations.

The background Newtonian time appears explicitly in the time-dependent Schrödinger equation (3), but it is pertinent to note that such a time is truly an abstraction in the sense that no *physical* clock can provide a precise measure of it [UW89]: there is always a small probability that a real clock will sometimes run backwards with respect to Newtonian time.

When we come to a  $\text{Diff}(M)$ -invariant theory like classical general relativity the role of time is very different. If  $M$  is equipped with a Lorentzian metric  $g$ , and if its topology is appropriate, it can be foliated in many ways as a one-parameter family of space-like surfaces, and each such parameter might be regarded as a possible definition of time. However several problems arise with this way of looking at things:

- There are many such foliations, and there is no way of selecting a particular one, or special family of such, that is ‘natural’ within the context of the theory alone.
- Such a definition of time is rather non-physical since it provides no hint as to how it might be measured or registered.

- The possibility of defining time in this way is closely linked to a fixed choice of the metric  $g$ . It becomes untenable if  $g$  is subject to some type of quantum fluctuation.

The last problem is crucial in any type-I approach to quantum gravity and raises a number of important questions. In particular:

- How *is* the notion of time to be incorporated in a quantum theory of gravity?
- Does it play a fundamental role in the construction of the theory or is it a ‘phenomenological’ concept that applies, for example, only in some coarse-grained, semi-classical sense?
- In the latter case, how reliable is the use at a basic level of techniques drawn from standard quantum theory?

The three main ways that have been suggested for solving the problem of time are as follows.

1. Fix some *background* causal structure and use that to determine temporal concepts in the quantum theory. Such a background might arise from two possible sources.
  - It might come from a *contingent* feature of the actual universe; for example, the 3<sup>0</sup>K thermal radiation. However, structure of this type is approximate and therefore works only if fine details are ignored. Also, there is a general matter of principle: do we expect a quantum theory of gravity to work for ‘all possible’ universes (whatever that might mean), or only for the actual one in which we happen to live?
  - An asymptotic causal structure could be associated with a spacetime manifold that is spatially non-compact and asymptotically flat. However, this would not help in the typical cosmological situation, and it is by no means obvious that ‘time’ defined in this way can be measured in any physically meaningful way.
2. Attempt to locate events both spatially and temporally with specific functionals of the gravitational and other fields. This important idea is based on the observation that, for example, if  $\phi$  is a scalar field then, as emphasised earlier, the value  $\phi(X)$  of  $\phi$  at a particular  $X \in M$  has no physical meaning. On the other hand, the value of  $\phi$  where something ‘is’ *does* have a physical meaning in the sense that ‘ $\phi(\text{thing})$ ’ is  $\text{Diff}(M)$  invariant.

The hope is that an ‘internal time’ of this type can be introduced in such a way that the normal dynamical equations of the classical theory are reproduced *precisely*. Then one would try to apply a similar technique to the quantum case. Ideas of this type have played a major role in the development of canonical quantum gravity.

3. The third approach starts by constructing some sort of quantum theory but with no reference to time at all. Physical time is then introduced as a reading on ‘real clocks’ but it is accepted that such a scheme will never *exactly* reproduce the

standard notion of time and that all physical clocks will at best work in some semi-classical limit. Approaches of this type are truly ‘timeless’ and raise the key issue of whether a meaningful quantum theory can indeed be created in a way that contains no fundamental reference to time. That this is not a trivial matter is exemplified by the remarks made earlier about the crucial role of time in conventional quantum theory.

### 3 Approaches to Quantum Gravity

As explained earlier, there are four general ways of trying to construct a quantum theory of gravity: I., quantize general relativity; II., general-relativise quantum theory; III., schemes constructed using standard quantum theory in which general relativity emerges only in some low-energy limit; and IV., schemes in which both general relativity and quantum theory emerge in some appropriate domain in the context of a theory that contains radical new ideas. Our task now is to see how the *prima facie* issues discussed in the previous section are addressed in some of these schemes.

#### 3.1 Quantize General Relativity

##### A. The Particle-Physics Approach

The early particle-physics based approaches to quantum gravity illustrate quite well a number of the issues discussed above. The starting point is to fix the background topology and differential structure of spacetime  $M$  to be that of Minkowski space, and then to write the Lorentzian metric  $g$  on  $M$  as

$$g_{\alpha\beta}(X) = \eta_{\alpha\beta} + h_{\alpha\beta}(X) \quad (4)$$

where  $h$  measures the departure of  $g$  from flat spacetime  $\eta$ .

The background metric  $\eta$  provides a fixed causal structure with the usual family of Lorentzian inertial frames. Thus, at this level, there is no problem of time. The causal structure also allows a notion of microcausality, thereby permitting a conventional type of relativistic quantum field theory to be applied to the field  $h_{\alpha\beta}$ . In particular, the quanta of this field (defined as usual using representations of the Poincaré group of isometries of  $\eta$ ) are massless spin-2 particles. A typical task would then be to compute perturbative scattering-matrix elements for these gravitons, both with each other and with the quanta of various matter fields. Note that there is no immediate problem of interpretation: the existence of a background spacetime manifold and causal structure fits in well with the standard Copenhagen view of quantum theory.

The action of  $\text{Diff}(M)$  is usually studied infinitesimally and is reflected in the quantum theory via a set of Ward identities that must be satisfied by the  $n$ -point functions of the theory. Thus the role of spacetime diffeomorphisms is also relatively straightforward.

It is clear that many of the *prima facie* issues discussed earlier are resolved in an approach of this type by virtue of its heavy use of background structure. However,

many classical relativists object violently to an expansion like (4), not least because the background <sup>2</sup> causal structure cannot generally be identified with the physical one. Also, one is restricted to a specific background topology, and so a scheme of this type is not well adapted for addressing many of the most interesting questions in quantum gravity: black-hole phenomena, quantum cosmology, phase changes *etc.* Nevertheless, if the scheme above had worked it would have been a major result and would undoubtedly have triggered a substantial effort to construct a covariant type-I theory in a non-perturbative way; a good analogue is the great increase in studies of lattice gauge theory that followed the proof by t'Hooft that Yang-Mills theory is perturbatively renormalisable.

Of course, this did not happen in the gravitational case because the ultraviolet divergences are sufficiently violent to render the theory perturbatively non-renormalisable. One reaction has been to regard this pathology as a result of using the expansion (4); an expansion that is, anyway, unpleasant when viewed from the canons of the classical theory. Several attempts have been made to construct a non-perturbative, covariant scheme, but none is particularly successful and it was only when Ashtekar made his important discoveries in the context of the *canonical* theory that the idea of non-perturbative quantisation really began to bear fruit.

The majority of particle physicists followed a different line and tried to enlarge the classical theory of general relativity with carefully chosen matter fields with the hope that the ultraviolet divergences would cancel, leaving a theory that is perturbatively well-behaved. The cancellation of a divergence associated with a loop of bosonic particles (like the graviton) can be achieved only by the introduction of *fermions*, and hence supergravity was born. However, supersymmetry requires very special types of matter, which supports the idea that a successful theory of quantum gravity must lead to a unified theory; *i.e.*, the extra fields needed to cancel the graviton infinities might be precisely those associated with some grand unified scheme of the fundamental forces.

Early expectations were high following successful low-order results but it is now generally accepted that if higher-loop calculations could be performed (they are very complex) intractable divergences would appear once more. However, this line of thinking is far from dead and the torch is currently carried by perturbative superstring theory.

Superstring theory has the great advantage over the simple covariant approaches that the individual terms in the appropriate perturbation expansion *can* be finite and, furthermore, the particle content of theories of this type could well be such as to relate the fundamental forces in a unified way.

The low-energy limit of these theories is a form of supergravity but, nevertheless, standard spacetime ideas do not play a very significant role. This is reflected by the graviton being only one of an infinite number of particles in the theory; similarly, the spacetime diffeomorphism group appears only as part of a much bigger structure. This down-playing of classical general relativity is typical of a type-III approach.

Notwithstanding the successes of superstring theory, some of the earlier objections to perturbative schemes still hold and, in addition, the superstring perturbation series

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<sup>2</sup> The scheme can be extended to use an arbitrary background metric, but this does not change the force of the objection.

is highly divergent. Hence much current attention is being devoted to the challenge of constructing a non-perturbative version of the theory. Most of the suggestions made so far work within the context of standard quantum theory and, in this sense, they are still of type III. However, the possibility also arises of finding a genuine type-IV structure whose low-energy limits would include standard quantum theory as well as supergravity.

### B. *The Canonical Approach*

The response to our *prima facie* questions given by the canonical approach to quantum gravity differs significantly from that of the particle-physics based schemes. Since my second lecture is devoted to canonical quantisation, I will merely sketch here some of the most important features.

1. *Use of standard quantum theory.* The basic technical ideas of standard quantum theory are employed, albeit adapted to handle the non-linear constraints satisfied by the canonical variables. On the other hand the traditional, Copenhagen type of interpretation of the theory is certainly *not* applicable in quantum cosmology, which is one of the most important potential uses of canonical quantum gravity.
2. *Background manifold structure.* The canonical theory of classical relativity assumes *ab initio* that the spacetime manifold  $M$  is diffeomorphic to  $\Sigma \times \mathbb{R}$  where  $\Sigma$  is some three-manifold. This three-manifold becomes part of the fixed background in the quantum theory and so, for example, there is no immediate possibility of discussing quantum topology.
3. *Background metric structure.* One of the main aspirations of the canonical approach to quantum gravity has always been to build a formalism with no background spatial, or spacetime, metric. This is particularly important in the context of quantum cosmology.
4. *The spacetime diffeomorphism group.* In the canonical form of general relativity the spacetime diffeomorphism group  $\text{Diff}(M)$  is replaced by a more complex entity (the ‘Dirac algebra’) which contains  $\text{Diff}(\Sigma)$  as a subgroup but which is not itself a genuine group. Invariance under  $\text{Diff}(\Sigma)$  means that the functionals of the canonical variables that correspond to physical variables are non-local with respect to  $\Sigma$ . The role of the full Dirac algebra is more subtle and varies according to which canonical scheme is followed.
5. *The problem of time.* In the absence of any background metric, this becomes a major issue. It is closely connected with the role of the Dirac algebra and the general question of what is meant by an ‘observable’. None of the several suggested ways for handling this problem work fully and it seems plausible that the standard concept of time can be recovered only in some semi-classical sense.
6. *The unification of the fundamental forces.* Perturbation theory in simple canonical quantum gravity is as badly defined mathematically as is its particle-physics based cousin. However, developments in the Ashtekar programme imply that it may be possible to construct a non-perturbative theory that is finite and that involves just the gravitational field alone. In this sense, canonical quantum gravity does *not* suggest that a unification of the forces is a necessary ingredient of a technically successful theory.

### 3.2 General-Relativise Quantum Theory

I cannot say much about the idea ‘general-relativising’ quantum theory (*i.e.*, type-II schemes) in relation to our *prima facie* issues because little research has been done in this area. A key role would probably be played by the spacetime diffeomorphism group  $\text{Diff}(M)$ : indeed, a type-II scheme might be *defined* as any attempt to force standard quantum theory to be compatible with  $\text{Diff}(M)$ ; of course the structure of  $M$  itself would then necessarily be part of the fixed background.

Important questions that arise in an approach of this type include:

- What form of quantum theory should be used? In particular, does it require a prior notion of ‘time’?
- What is the role of the field equations of classical general relativity? Do they also need to be imposed as part of the structural background, or is the fundamental input the spacetime diffeomorphism group alone?
- Is there a ‘canonical’ version in which standard quantum theory is forced to be compatible with the Dirac algebra rather than with  $\text{Diff}(M)$ ?
- There is a long-standing and extensive research programme to construct quantum field theories (usually linear) in a background spacetime manifold equipped with a fixed Lorentzian metric. Does this work throw any light on the idea of general-relativising quantum theory?

### 3.3 The Use of Radical New Concepts

In the classification we have been using, a type-IV scheme is any approach to quantum gravity that starts with a view of quantum theory and spacetime physics that is radically different from that of conventional theories, and with the expectation that these standard ideas will emerge only in some limited domain. Almost by definition, schemes of this type dispense with much of the background structure of other approaches to quantum gravity, including, possibly, standard quantum theory as well as many normal spacetime concepts. Unfortunately, such schemes tend to be individualistic in form, and their manner of dealing with our *prima facie* issues has to be treated on a strictly *ad hominum* basis.

## 4 Conclusions

The two major current approaches to quantum gravity proper—the Ashtekar programme, and superstring theory—differ so much in their starting positions and lines of development that it is hard to say much in conclusion other than that the problem of quantum gravity is still wide open. In particular, and *pace* the discussion above, the jury is still out on the all-important question of whether a consistent theory of quantum gravity can

be achieved within the framework of our existing understanding of physics, or whether some radical change is needed before any real headway can be made.

The problem of time is crucial in this respect, and its resolution is still very unclear. However, one reading of the current situation is that normal notions of time and space are applicable only at scales well above the Planck regime. If true, such a position throws great doubt on the use of *any* standard quantum ideas as a basic ingredient in the theory; indeed, a more plausible scenario is that standard quantum theory becomes applicable at precisely the same point in the formalism as does the normal notion of time. Such a situation is exciting for those who, like myself, enjoy indulging in speculative metaphysics/theoretical-physics, but it is also most frustrating in the absence of any clear empirical data that could point us in the right direction. The problem of quantum gravity continues to be a challenge for the next century!

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