

Strings, loops and others:
a critical survey of the present approaches to quantum gravity
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Abstract

I review the present theoretical attempts to understand the quantum properties of spacetime. In particular, I illustrate the main achievements and the main difficulties in: string theory, loop quantum gravity, discrete quantum gravity (Regge calculus, dynamical triangulations and simplicial models), Euclidean quantum gravity, perturbative quantum gravity, quantum field theory on curved spacetime, noncommutative geometry, null surfaces, topological quantum field theories and spin foam models. I also briefly review several recent advances in understanding black hole entropy and attempt a critical discussion of our present understanding of quantum spacetime.

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I. INTRODUCTION

The landscape of fundamental physics has changed substantially during the last one or two decades. Not long ago, our understanding of the weak and strong interactions was very confused, while general relativity was almost totally disconnected from the rest of physics and was empirically supported by little more than its three classical tests. Then two things have happened. The $SU(3) \times SU(2) \times U(1)$ Standard Model has found a dramatic empirical success, showing that quantum field theory (QFT) is capable of describing all accessible fundamental physics, or at least all non-gravitational physics. At the same time, general relativity (GR) has undergone an extraordinary “renaissance”, finding widespread application in astrophysics and cosmology, as well as novel vast experimental support – so that today GR is basic physics needed for describing a variety of physical systems we have access to, including, as have heard in this conference, advanced technological systems [1].

These two parallel developments have moved fundamental physics to a position in which it has rarely been in the course of its history: We have today a group of fundamental laws, the standard model and GR, which –even if it cannot be regarded as a satisfactory global picture of Nature– is perhaps the best confirmed set of fundamental theories after Newton’s universal gravitation and Maxwell’s electromagnetism. More importantly, there aren’t today experimental facts that openly challenge or escape this set of fundamental laws. In this unprecedented state of affairs, a large number of theoretical physicists from different backgrounds have begun to address *the* piece of the puzzle which is clearly missing: combining the two halves of the picture and understanding the quantum properties of the gravitational field. Equivalently, understanding the quantum properties of spacetime. Interest and researches in quantum gravity have thus increased sharply in recent years. And the problem of understanding what is a quantum spacetime is today at the core of fundamental physics.

The problem is not anymore in the sole hands of the relativists. A large fraction of the particle physicists, after having mostly ignored gravity for decades, are now exploring issues such as black hole entropy, background independence, and the Einstein equations. Today, in both the gr-qc and hep-th sectors of the Los Alamos archives, an average of one paper every four is related to quantum gravity, a much higher proportion than anytime before.

This sharp increase in interest is accompanied by some results. First of all, we do have today some well developed and reasonably well defined tentative theories of quantum gravity. String theory and loop quantum gravity are the two major examples. Second, within these theories definite physical results have been obtained, such as the explicit computation of the “quanta of geometry” and the derivation of the black hole entropy formula. Furthermore, a number of fresh new ideas –for instance, non-

commutative geometry– have entered quantum gravity.

A lot of activity does not necessarily mean that the solution has been reached, or that it is close. There is a lot of optimism around, but the optimism is not shared by everybody. In particular, in recent years we have repeatedly heard, particularly from the string camp, bold claims that we now *have* a convincing and comprehensive theory of Nature, including the solution of the quantum gravity puzzle. But many think that these claims are not substantiated. So far, no approach to quantum gravity can claim even a single piece of experimental evidence. In science a theory becomes credible only after corroborated by experiments – since then, it is just an hypothesis – and history is full of beautiful hypotheses later contradicted by Nature. The debate between those who think that string theory is clearly the correct solution and those who dispute this belief is a major scientific debate, and one of the most interesting and stimulating debates in contemporary science. This work is also meant as a small contribution to this debate.

But if an excess of confidence is, in the opinion of many, far premature, a gloomy pessimism, also rather common, is probably not a very productive attitude either. The recent explosion of interest in quantum gravity has led to some progress and *might* have taken us much closer to the solution of the puzzle. In the last years the main approaches have obtained theoretical successes and have produced predictions that are at least testable in principle, and whose indirect consequences are being explored.

One does not find if one does not search. The search for understanding the deep quantum structure of spacetime, and for a conceptual framework within which everything we have learned about the physical world in this century could stay together consistently, is so fascinating and so intellectually important that it is worthwhile pursuing even at the risk of further failures. The research in quantum gravity in the last few years has been vibrant, almost in fibrillation. I will do my best to give an overview of what is happening. In the next sections, I present an overview of the main present research direction, a discussion of the different perspectives in which the problem of quantum gravity is perceived by the different communities addressing it, and a tentative assessment of the achievements and the state of the art.

I have done my best to reach a balanced view, but the field is far from a situation in which consensus has been reached, and the best I can offer, of course, is my own biased perspective. For a previous overview of the problem of quantum gravity, see [2].

Traditional	Most Popular	New
Discrete Dynamical triangulations Regge calculus Simplicial models → 2^{nd} order transition?	Strings → <i>Black hole entropy</i>	Noncommutative geometry → <i>Quantum theory?</i>
Approximate theories Euclidean quantum gravity Perturbative q.g. → <i>Woodard-Tsamis effect</i> QFT on curved spacetime	Loops → <i>Black hole entropy</i> → <i>Eigenvalues of the geometry:</i> $A_{\vec{j}} = 8\pi\hbar G \sum_i \sqrt{j_i(j_i + 1)}$	Null surfaces → <i>Observables?</i>
Unorthodox Sorkin's Posets Finkelstein Twistors ...		Spin foam models → <i>convergence of loop, discrete, TQFT and sum-over-histories</i>

Main current approaches to quantum gravity

II. DIRECTIONS

To get an idea of what the community is working on, I have made some amateurish statistical analysis of the subjects of the papers in the Los Alamos archives. The archives which are particularly relevant for quantum gravity are gr-qc and hep-th. The split between the two reflects quite accurately the two traditions, or the two cultures, that are now addressing the problem. hep-th is almost 3 times larger than gr-qc: 295 versus 113 papers per month – average over last year. In each of the two archives, roughly 1/4 of the papers relate to quantum gravity. Here is a breaking up of these paper per field, in an average month:

String theory:	69
Loop quantum gravity:	25
QFT in curved spaces:	8
Lattice approaches:	7
Euclidean quantum gravity:	3
Non-commutative geometry:	3
Quantum cosmology:	1
Twistors:	1
Others:	6

Most of the string papers are in hep-th, most of the others are in gr-qc. These data confirm two ideas: that issues related to quantum gravity occupy a large part of contemporary theoretical research in fundamental physics, and that the research is split into two camps.

There are clearly two most popular approaches to quantum gravity: a major one, string theory, popular among particle physicists, and a (distant) second, loop quantum gravity, popular among relativists. String theory [3] can be seen as the natural outcome of the line of research that started with the effort to go beyond the standard model, and went through grand unified theories, supersymmetry and supergravity. Loop quantum gravity [4] can be seen as the natural outcome of the line of research that started with Dirac's interest in quantizing gravity, which led him to the development of the theory of the quantization of constrained systems; and continued with the construction of canonical general relativity by Dirac himself, Bergmann, Arnowit Deser and Misner, the pioneering work in quantum gravity of John Wheeler and Bryce DeWitt, and the developments of this theory by Karel Kuchar, Chris Isham and many others.

String theory and loop quantum gravity are characterized by surprising similarities (both are based on one-dimensional objects), but also by a surprising divergence in philosophy and results. String theory defines a superb “low” energy theory, but finds difficulties in describing

Planck scale quantum spacetime directly. Loop quantum gravity provides a beautiful and compelling account of Planck scale quantum spacetime, but finds difficulties in connecting to low energy physics.

Besides strings and loops, a number of other approaches are being investigated. A substantial amount of energy has been recently devoted to the attempt of defining quantum gravity from a discretization of general relativity, on the model of lattice QCD (Dynamical triangulations, quantum Regge calculus, simplicial models). A number of approaches (Euclidean quantum gravity, old perturbative quantum gravity, quantum field theory on curved spacetime) aim at describing certain regimes of the quantum behavior of the gravitational field in approximate form, without the ambition of providing the fundamental theory, even if they previously had greater ambitions. More “outsider” and radical ideas, such as twistor theory, Finkelstein’s algebraic approach, Sorkin’s poset theory, continue to raise interest. Finally, the last years have seen the appearance of radically new ideas, such as noncommutative geometry, the null surface formulation, and spin foam models. I have summarized the main approaches in the Table above.

The various approaches are far from independent. There are numerous connections and there is convergence and cross-fertilization in ideas, techniques and results.

III. MAIN DIRECTIONS

A. String theory

String theory is by far the research direction which is presently most investigated. I will not say much about the theory, which was covered in this conference in the plenary lecture by Gary Gibbons. I will only comment on the relevance of string theory for the problem of understanding the quantum properties of spacetime. String theory presently exists at two levels. First, there is a well developed set of techniques that define the string perturbation expansion over a given metric background. Second, the understanding of the nonperturbative aspects of the theory has much increased in recent years [5] and in the string community there is a widespread faith, supported by numerous indications, in the existence of a yet-to-be-found full non-perturbative theory, capable of generating the perturbation expansion. There are attempts of constructing this non-perturbative theory, generically denoted M theory. The currently popular one is Matrix-theory, of which it is far too early to judge the effectiveness [6].

The claim that string theory solves QG is based on two facts. First, the string perturbation expansion includes the graviton. More precisely, one of the string modes is a massless spin two, and helicity ± 2 , particle. Such a particle necessarily couples to the energy-momentum

tensor of the rest of the fields [7] and gives general relativity to a first approximation. Second, the perturbation expansion is consistent if the background geometry over which the theory is defined satisfies a certain consistency condition; this condition turns out to be a high energy modification of the Einstein’s equations. The hope is that such a consistency condition for the perturbation expansion will emerge as a full-fledged dynamical equation from the yet-to-be-found nonperturbative theory.

From the point of view of the problem of quantum gravity, the relevant physical results from string theory are two.

Black hole entropy. The most remarkable physical results for quantum gravity is the derivation of the Bekenstein-Hawking formula for the entropy of a black hole (See Section VI) as a function of the horizon area. This beautiful result has been obtained last year by Andy Strominger and Cumrun Vafa [8], and has then been extended in various directions [9–13]. The result indicates that there is some unexpected internal consistency between string theory and QFT on curved space. In section VI, I illustrate this result in some detail and I compare it with similar results recently obtained in other approaches.

Microstructure of spacetime. There are indications that in string theory the spacetime continuum is meaningless below the Planck length. An old set of results on very high energy scattering amplitudes [14] indicates that there is no way of probing the spacetime geometry at very short distances. What happens is that in order to probe smaller distance one needs higher energy, but at high energy the string “opens up from being a particle to being a true string” which is spread over spacetime, and there is no way of focusing a string’s collision within a small spacetime region.

More recently, in the Matrix-theory nonperturbative formulation [6], the space-time coordinates of the string x^i are replaced by matrices $(X^i)_m^n$. This can perhaps be viewed as a new interpretation of the space-time structure. The continuous space-time manifold emerges only in the long distance region, where these matrices are diagonal and commute; while the space-time appears to have a non-commutative discretized structure in the short distance regime. These features are still poorly understood, but they have intriguing resonances with noncommutative geometry [15] (Section V A) and loop quantum gravity (Section III B).

1. Difficulties with string theory

A key difficulty in string theory is the lack of a complete nonperturbative formulation. During the last year,

there has been excitement for some tentative nonperturbative formulations [6]; but it is far too early to understand if these attempts will be successful. Many previously highly acclaimed ideas have been rapidly forgotten.

A distinct and even more serious difficulty of string theory is the lack of a *background independent* formulation of the theory. In the words of Ed Witten:

“Finding the right framework for an intrinsic, background independent formulation of string theory is one of the main problems in string theory, and so far has remained out of reach.” ... “This problem is fundamental because it is here that one really has to address the question of what kind of geometrical object the string represents.” [16]

Most of string theory is conceived in terms of a theory describing excitations over this or that background, possibly with connections between different backgrounds. This is also true for (most) nonperturbative formulations such as Matrix theory. For instance, the (bosonic part of the) lagrangian of Matrix-theory that was illustrated in this conference by Gary Gibbons is

$$L \sim \frac{1}{2} \text{Tr} \left(\dot{X}^2 + \frac{1}{2} [X^i, X^j]^2 \right). \quad (1)$$

The indices that label the matrices X^i are raised and lowered with a Minkowski metric, and the theory is Lorentz invariant. In other words, the lagrangian is really

$$L \sim \frac{1}{2} \text{Tr} \left(g^{00} g^{ij} \dot{X}_i \dot{X}_j + \frac{1}{2} g^{ik} g^{jl} [X_i, X_j] [X_k, X_l] \right), \quad (2)$$

where g is the flat metric of the background. This shows that there is a non-dynamical metric, and an implicit flat background in the action of the theory. (For attempts to explore background independent Matrix-theory, see [117]).

But the world is not formed by a fixed background over which things happen. The background itself is dynamical. In particular, for instance, the theory should contain quantum states that are quantum superpositions of different backgrounds – and presumably these states play an essential role in the deep quantum gravitational regime, namely in situations such as the big bang or the final phase of black hole evaporation. The absence of a fixed background in nature (or active diffeomorphism invariance) is the key general lessons we have learned from gravitational theories. I discuss this issue in more detail in section VII. In the opinion of many, until string theory finds a genuine background independent formulations, it will never have a convincing solution of the quantum gravity puzzle. Until string theory describes excitations located over a metric background, the central problem of a true merge of general relativity and quantum mechanics, and of understanding quantum space-time, has not been addressed.

Finally there isn’t any direct or indirect experimental support for string theory (as for any other approach to quantum gravity). Claiming, as it is sometimes done, that a successful physical prediction of string theory is GR is a nonsense for various reasons. First, by the same token one could claim that the $SU(5)$ grand unified theory (an extremely beautiful theoretical idea, sadly falsified by the proton decay experiments) is confirmed by the fact that it predicts electromagnetism. Second, GR did not emerge as a surprise from string theory: it is because string theory could describe gravity that it was taken seriously as a unified theory. Third, if GR was not known, nobody would have thought of replacing the flat spacetime metric in the string action with a curved and dynamical metric. “Predicting” a spin-two particle is no big deal in a theory that predicts any sort of still unobserved other particles. The fact that string theory includes GR is a necessary condition for taking it seriously as a promising tentative theory of quantum gravity, not an argument in support of its physical correctness.

An important remark is due in this regard. All the testable predictions made after the standard model, such as proton decay, monopoles, existence of supersymmetric partners, exotic particles ..., have, so far, *all* failed to be confirmed by time and money consuming experiments designed to confirm them. The comparison between these failed predictions and the extraordinary confirmation obtained by *all* the predictions of the standard model (neutral currents, W and Z particles, top quark ...) may contain a lesson that should perhaps make us reflect. If all predictions are confirmed until a point, and all predictions fail to be confirmed afterwards, one might suspect that a wrong turn might have been taken at that point. Contrary to what sometimes claimed, the theoretical developments that have followed the standard model, such as for instance supersymmetry, are *only* fascinating but non-confirmed *hypotheses*. As far we really know, nature may very well have chosen otherwise.

Experimental observation of supersymmetry might very well change this balance, and may be in close reach. But we have been thinking that observation of supersymmetry was around the corner for quite sometime now, and it doesn’t seem to show up yet. Until it does, if there is any indication at all coming from the experiments, this indication is that all the marvelous ideas that followed the standard model may very well be all in the wrong direction. The great tragedy of science, said TH Huxley, is the slaying of a beautiful hypothesis by brute facts.

In spite of these difficulties, string theory is today, without doubt, the leading and most promising candidate for a quantum theory of gravity. It is the theory most studied, most developed and closer to a comprehensive and consistent framework. It is certainly extremely beautiful, and the recent derivation of the black hole entropy formula with the exact Bekenstein-Hawking coefficient represents a definite success, showing that the understanding of the theory is still growing.

2. String cosmology

There has been a burst of recent activity in an outgrowth of string theory denoted string cosmology [17]. The aim of string cosmology is to extract physical consequences from string theory by applying it to the big bang. The idea is to start from a Minkowski flat universe; show that this is unstable and therefore will run away from the flat (false-vacuum) state. The evolution then leads to a cosmological model that starts off in an inflationary phase. This scenario is described using minisuperspace technology, in the context of the low energy theory that emerge as limit of string theory. Thus, first one freezes all the massive modes of the string, then one freezes all massless modes except the zero modes (the spatially constant ones), obtaining a finite dimensional theory, which can be quantized non-perturbatively. The approach has a puzzling aspect, and a very attractive aspect.

The puzzling aspect is its overall philosophy. Flat space is nothing more than an accidental local configuration of the gravitational field. The universe as a whole has no particular sympathy for flat spacetime. Why should we consider a cosmological model that begins with a flat spacetime? To make this point clear using a historical analogy, string cosmology is a little bit as if after Copernicus discovered that the Earth is not in the center of the solar system, somebody would propose the following explanation of the *birth* of the solar system: at the beginning the Earth was in the center. But this configuration is unstable (it is!), and therefore it decayed into another configuration in which the Earth rotates around the Sun. This discussion emphasizes the profound cultural divide between the relativity and the particle physicists' community, in dealing with quantum spacetime.

The compelling aspect of string cosmology, on the other hand, is that it provides a concrete physical application of string theory, which might lead to consequences that are in principle observable. The spacetime emerging from the string cosmology evolution is filled with a background of gravitational waves whose spectrum is constrained by the theory. It is not impossible that we will be able to measure the gravitational wave background not too far in the future, and the prospect of having a way for empirically testing a quantum gravity theory is very intriguing.

B. Loop quantum gravity

The second most popular approach to quantum gravity, and the most popular among relativists is loop quantum gravity [4]. Loop quantum gravity is presently the best developed alternative to string theory. Like strings, it is not far from a complete and consistent theory and it yields a corpus of definite physical predictions, testable in principle, on quantum spacetime.

Loop quantum gravity, however, attacks the problem from the opposite direction than string theory. It is a non-perturbative and background independent theory to start with. In other words, it is deeply rooted into the conceptual revolution generated by general relativity. In fact, successes and problems of loop quantum gravity are complementary to successes and problems of strings. Loop quantum gravity is successful in providing a consistent mathematical and physical picture of non perturbative quantum spacetime; but the connection to the low energy dynamics is not yet completely clear.

The general idea on which loop quantum gravity is based is the following. The core of quantum mechanics is not identified with the structure of (conventional) QFT, because conventional QFT presupposes a background *metric* spacetime, and is therefore immediately in conflict with GR. Rather, it is identified with the general structure common to all quantum systems. The core of GR is identified with the absence of a fixed observable background spacetime structure, namely with active diffeomorphism invariance. Loop quantum gravity is thus a quantum theory in the conventional sense: a Hilbert space and a set of quantum (field) operators, with the requirement that its classical limit is GR with its conventional matter couplings. But it is not a QFT over a metric manifold. Rather, it is a “quantum field theory on a differentiable manifold”, respecting the manifold's invariances and where only coordinate independent quantities are physical.

Technically, loop quantum gravity is based on two inputs:

- The formulation of classical GR based on the Ashtekar connection [18]. The version of the connection now most popular is not the original complex one, but an evolution of the same, in which the connection is real.
- The choice of the holonomies of this connection, denoted “loop variables”, as basic variables for the quantum gravitational field [19].

This second choice determines the peculiar kind of quantum theory being built. Physically, it corresponds to the assumption that excitations with support on a loop are normalizable states. This is the key technical assumption on which everything relies.

It is important to notice that this assumption fails in conventional 4d Yang Mills theory, because loop-like excitations on a metric manifold are too singular: the field needs to be smeared in more dimensions. Equivalently, the linear closure of the loop states is a “far too big” non-separable state space. This fact is the major source of some particle physicists's suspicion at loop quantum gravity. What makes GR different from 4d Yang Mills theory, however, is nonperturbative diffeomorphism invariance. The gauge invariant states, in fact, are not localized at all – they are, pictorially speaking, smeared

by the (gauge) diffeomorphism group all over the coordinates manifold. More precisely, factoring away the diffeomorphism group takes us down from the state space of the loop excitations, which is “too big”, to a separable physical state space of the right size [24,21]. Thus, the consistency of the loop construction relies heavily on diffeomorphism invariance. In other words, the diffeomorphism invariant loop states (more precisely, the diffeomorphism invariant spin network states) are not physical excitations of a field *on* spacetime. They are excitations *of* spacetime itself.

Loop quantum gravity is today ten years old. Actually, the first announcement of the theory was made in India precisely 10 years ago, *today!* [22] In the last years, the theory has grown substantially in various directions, and has produced a number of results, which I now briefly illustrate.

Definition of theory. The mathematical structure of the theory has been put on a very solid basis. Early difficulties have been overcome. In particular, there were three major problems in the theory: the lack of a well defined scalar product, the overcompleteness of the loop basis, and the difficulty of treating the reality conditions.

- The problem of the lack of a scalar product on the Hilbert space has been solved with the definition of a diffeomorphism invariant measure on a space of connections [23]. Later, it has also become clear that the same scalar product can be defined in a purely algebraic manner [24]. The state space of the theory is therefore a genuine Hilbert space $\mathcal{H}$.
- The overcompleteness of the loop basis has been solved by the introduction of the spin network states [25]. A spin network is a graph carrying labels (related to $SU(2)$ representations and called “colors”) on its links and its nodes.

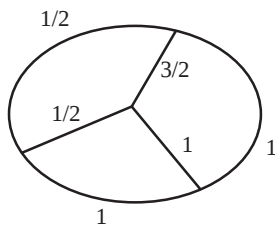


FIG. 1. Figure 1: A simple spin network. Only the coloring of the links is indicated.

Each spin network defines a spin network state, and the spin network states form a (genuine, non-overcomplete) orthonormal basis in $\mathcal{H}$.

- The difficulties with the reality conditions have been circumvented by the use of the real formulation [26,27].

The kinematics of loop quantum gravity is now defined with a level of rigor characteristic of mathematical physics [29] and the theory can be defined using various alternative techniques [24,30].

Hamiltonian constraint. A rigorous definition version of the hamiltonian constraint equation has been constructed [28]. This is anomaly free, in the sense that the constraints algebra closes (but see later on). The hamiltonian has the crucial properties of acting on nodes only, which implies that its action is naturally discrete and combinatorial [19,31]. This fact is at the roots of the existence of exact solutions [19,32], and of the possible finiteness of the theory

Matter. The old hope that QFT divergences could be cured by QG has recently received an interesting corroboration. The matter part of the hamiltonian constraint is well-defined without need of renormalization [33]. Thus, a main possible stumbling block is over: infinities did not appear in a place where they could very well have appeared.

Physical Results.

Black hole entropy. The first important physical result in loop quantum gravity is a computation of black hole entropy [102–104]. I describe this result and I compare it with other derivations in section VI.

Quanta of geometry. A very exciting development in quantum gravity in the last years has been by the computations of the quanta of geometry. That is, the computation of the discrete eigenvalues of area and volume. I describe this result a bit more in detail in the next section.

1. Quanta of Geometry

In quantum gravity, any quantity that depends on the metric becomes an operator. In particular, so do the area A of a given (physically defined) surface, or the volume V of a given (physically defined) spatial region. In loop quantum gravity, these operators can be written explicitly. They are mathematically well defined self-adjoint operators in the Hilbert space $\mathcal{H}$. We know from quantum mechanics that certain physical quantities are quantized, and that we can compute their discrete values by computing the eigenvalues of the corresponding operator. Therefore, if we can compute the eigenvalues of the area and volume operators, we have a physical prediction on the possible quantized values that these quantities can take, at the Planck scale. These eigenvalues have been computed in loop quantum gravity [37]. Here is for instance the main sequence of the spectrum of the area

$$A_{\vec{j}} = 8\pi\gamma\hbar G \sum_i \sqrt{j_i(j_i + 1)}. \quad (3)$$

$\vec{j} = (j_1, \dots, j_n)$ is an n -tuple of half-integers, labeling the eigenvalues, G and $\hbar$ are the Newton and Planck constants, and γ is a dimensionless free parameter, denoted the Immirzi parameter, not determined by the theory [105,106]. A similar result holds for the volume. The spectrum (3) has been rederived and completed using various different techniques [24,38,39].

These spectra represent solid results of loop quantum gravity. Under certain additional assumptions on the behavior of area and volume operators in the presence of matter, these results can be interpreted as a corpus of detailed quantitative predictions on hypothetical Planck scale observations. Eq.(3) plays a key role also in black hole physics; see Section VI.

Besides its direct relevance, the quantization of the area and the volume is of interest because it provides a physical picture of quantum spacetime. The states of the spin network basis are eigenstates of some area and volume operators. We can say that a spin network carries quanta of area along its links, and quanta of volume at its nodes. The magnitude of these quanta is determined by the coloring. For instance, the half-integers $j_1 \dots j_n$ in (3) are the coloring of the spin network's links that cross the given surface. Thus, a quantum spacetime can be decomposed in a basis of states that can be visualized as made by quanta of volume (the intersections) separated by quanta of area (the links). More precisely, we can view a spin network as sitting on the *dual* of a cellular decomposition of physical space. The nodes of the spin network sit in the center of the 3-cells, and their coloring determines the (quantized) 3-cell's volume. The links of the spin network cut the faces of the cellular decomposition, and their color $\vec{j}$ determine the (quantized) areas of these faces via equation (3). See Figure 2.

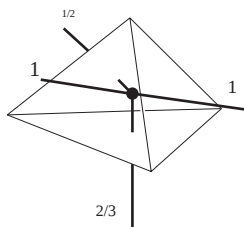


FIG. 2. Figure 2: A node of a spin network (in bold) and its dual 3-cell (here a tetrahedron). The coloring of the node determines the quantized volume of the tetrahedron. The coloring of the links (shown in the picture) determines the quantized area of the faces via equation (3). Here the vector $\vec{j}$ has a single component, because each face is crossed by one link only.

Finally, a recent evolution in loop quantum gravity looks particularly promising. A spacetime, path integral-like, formulation theory has been derived from the canonical theory. This evolution represents the merging between loop quantum gravity and other research direc-

tions; I illustrate it in section V C 2.

2. Difficulties with loop quantum gravity

While the kinematics of quantum spacetime is well understood, its dynamics is much less clear. The main problem originates from the quantum constraint algebra. The algebra is anomaly free, in the sense that it closes. However, it differs from the classical one in a subtle sense [40]. For this reason and others [41], doubts have been raised on the correctness of the proposed form of the hamiltonian constraint, and variants have been considered [92,41]. A solid proof that any of these versions yields classical GR in the classical limit, however, is lacking.

Furthermore, a systematic way of extracting physical prediction from the theory, analogous, say, to the perturbative QFT scattering expansion, is not yet available. Finally, a description of the Minkowski vacuum state is notably absent. Thus, the theory describe effectively quantum spacetime, but the extent to which low energy physics can be recovered is unclear.

In summary, the mathematics of the theory is solidly defined and understood from alternative points of view. Longstanding problems (lack of a scalar product, over-completeness of the loop basis and reality condition) have been solved. This kinematics provides a compelling description of quantum spacetime in terms of discrete excitations of the geometry carrying discretized quanta of area and volume. The theory can be extended to include matter, and there are strong indications that ultraviolet divergences do not appear. A spacetime covariant version of the theory, in the form of a topological sum over surfaces is under development. The main physical results derived so far are the computation of the eigenvalues of area and volume, and the derivation of the black hole entropy formula. Version of the dynamics exists, but a proof that the classical limit is classical GR is still lacking. The main open problems are to determine the correct version of the hamiltonian constraint and to understand how to describe the low energy regime.

IV. TRADITIONAL APPROACHES

A. Discrete approaches

Discrete quantum gravity is the program of regularizing classical GR in terms of some lattice theory, quantize this lattice theory, and then study an appropriate continuum limit, as one may do in QCD. There are three main ways of discretizing GR.

Regge introduced the idea of triangulating spacetime by means of a simplicial complex and using the lengths l_i of the links of the complex as gravitational variables [42]. The theory can then be quantized by integrating over the lengths l_i of the links. For a recent review and extensive references see [43]. Recent work has focused in problems such as the geometry of Regge superspace [44] and choice of the integration measure [45]. Some difficulties of this approach have recently been discussed in [46], where it is claimed that quantum Regge calculus fails to reproduce the results obtained in the continuum in the lower dimensional cases where the continuum theory is known.

2. Dynamical triangulations

Alternatively, one can keep the length of the links fixed, and capture the geometry by means of the way in which the simplices are glued together, namely by the triangulation. The Einstein-Hilbert action of Euclidean gravity is approximated by a simple function of the total number of simplices and links, and the theory can be quantized summing over distinct triangulations. For a detailed introduction, a recent review, and all relevant references [and, last but not least, Mauro Carfora's (and Gaia's!) drawings], see [47]. There are two coupling constants in the theory, roughly corresponding to the Newton and cosmological constants. These define a two dimensional space of theories. The theory has a nontrivial continuum limit if in this parameter space there is a critical point corresponding to a second order phase transition. The theory has phase transition and a critical point [48]. The transition separates a phase with crumpled spacetimes from a phase with "elongated" spaces which are effectively two-dimensional, with characteristic of a branched polymer [49,50]. This polymer structure is surprisingly the same as the one that emerges from loop quantum gravity at short scale. Near the transition, the model appears to produce "classical" S^4 spacetimes, and there is evidence for scaling, suggesting a continuum behavior [49]. However, evidence has been contradictory on whether or not the critical point is of the second order, as required for a nontrivial scaling limit. The consensus seems to be clustering for a first order transition [51]. This could indicate that the approach does not lead to a continuum theory.

Ways out from this serious impasse are possible. First, it has been suggested that even a first order phase transition may work in this context [52]. Second, Brügmann and Marinari have noticed that there is some freedom in the definition of the measure in the sum over triangulations, and have suggested (before the transition was shown to be first order) that taking this into account might change the nature of the transition [53].

A third road for discretizing GR was opened by a celebrated paper by Ponzano and Regge [54]. Ponzano and Regge started from a Regge discretization of three-dimensional GR and introduced a second discretization, by posing the ansatz (the *Ponzano Regge ansatz*) that the lengths l assigned to the links are discretized as well, in half-integers in Planck units

$$l = \hbar G j, \quad j = 0, \frac{1}{2}, 1, \dots \quad (4)$$

(Planck length is $\hbar G$ in 3d.) The half integers j associated to the links are denoted "coloring" of the triangulation. Coloring can be viewed as the assignment of a $SU(2)$ irreducible representation to each link of the Regge triangulation. The elementary cells of the triangulation are tetrahedra, which have six links, colored with six $SU(2)$ representations. $SU(2)$ representation theory naturally assigns a number to a sextuplet of representations: the Wigner 6- j symbol. Rather magically, the product over all tetrahedra of these 6- j symbols converges to (the real part of the exponent of) the Einstein Hilbert action. Thus, Ponzano and Regge were led to propose a quantization of 3d GR based on the partition function

$$Z \sim \sum_{\text{coloring}} \prod_{\text{tetrahedra}} 6-j(\text{color of the tetrahedron}) \quad (5)$$

(I have neglected some coefficients for simplicity). They also provided arguments indicating that this sum is independent from the triangulation of the manifold.

The formula (5) is simple and beautiful, and the idea has recently had many surprising and interesting developments. Three-dimensional GR was quantized as a topological field theory (see Section VC 1) in [55] and using loop quantum gravity in [56]. The Ponzano-Regge quantization based on equation (5) was shown to be essentially equivalent to the TQFT quantization in [86], and to the loop quantum gravity in [58]. (For an extensive discussion of quantum gravity in 3 dimensions and what we have learned from it, see [59].)

Something remarkable happens in establishing the relation between the Ponzano-Regge approach and the loop approach: the Ponzano-Regge ansatz (4) can be *derived* from loop quantum gravity [58]. Indeed, (4) turns out to be nothing but the 2d version of the 3d formula (3), which gives the quantization of the area. Therefore, a key result of quantum gravity of the last years, namely the quantization of the geometry, derived in the loop formalism from a full fledged nonperturbative quantization of GR, was anticipated as an ansatz by the intuition of Ponzano and Regge.

Surprises continued with the attempts to extend these ideas to 4 dimensions. These attempts have lead to a fascinating convergence of discrete gravity, topological

quantum field theory, and loop quantum gravity. I describe these developments in section VC2. I only notice here that the connection between the Ponzano-Regge ansatz and the quantization of the length in 3d loop gravity indicates immediately that in 4 spacetime dimensions the naturally quantized geometrical quantities are not the lengths of the links, but rather areas and volumes of 2-cells and 3-cells of the triangulation [58,60]. Therefore the natural coloring of the 4 dimensional state sum models should be on the 2-cells and 3-cells. We will see in section VC2 that this is precisely the case.

B. Old hopes → approximate theories

1. Euclidean quantum gravity

Euclidean quantum gravity is the approach based on a formal sum over Euclidean geometries

$$Z \sim \mathcal{N} \int \mathcal{D}[g] e^{-\int d^4x \sqrt{g} R[g]}. \quad (6)$$

As far as I understand, Hawking and his close collaborators do not anymore view this approach as an attempt to directly define a fundamental theory. The integral is badly ill defined, and does not lead to any known viable perturbation expansion. However, the main ideas of this approach are still alive in several ways.

First, Hawking’s picture of quantum gravity as a sum over spacetimes continues to provide a powerful intuitive reference point for most of the research related to quantum gravity. Indeed, many approaches can be seen as attempts to replace the ill defined and non-renormalizable formal integral (6) with a well defined expression. The dynamical triangulation approach (Section IV A) and the spin foam approach (Section VC2) are examples of attempts to realize Hawking’s intuition. Influence of Euclidean quantum gravity can also be found in the Atiyah axioms for TQFT (Section VC1).

Second, this approach can be used as an approximate method for describing certain regimes of nonperturbative quantum spacetime physics, even if the fundamental dynamics is given by a more complete theory. In this spirit, Hawking and collaborators have continued the investigation of phenomena such as, for instance, pair creation of black holes in a background de Sitter spacetime. Hawking and Bousso, for example, have recently studied the evaporation and “anti-evaporation” of Schwarzschild-de Sitter black holes [61].

2. Perturbative quantum gravity as effective theory, and the Woodard-Tsamis effect

If expand classical GR around, say, the Minkowski metric, $g_{\mu\nu}(x) = \eta_{\mu\nu} + h_{\mu\nu}(x)$, and construct a conventional QFT for the field $h_{\mu\nu}(x)$, we obtain, as it is well

known, a non renormalizable theory. A small but intriguing group of papers has recently appeared, based on the proposal of treating this perturbative theory seriously, as a respectable low energy effective theory by its own. This cannot solve the deep problem of understanding the world in general relativistic quantum terms. But it can still be used for studying quantum properties of spacetime in some regimes. This view has been advocated in a convincing way by John Donoghue, who has developed effective field theory methods for extracting physics from non renormalizable quantum GR [63].

In this spirit, a particularly intriguing result is presented in a recent work by RP Woodard and NC Tsamis [62]. Woodard and Tsamis consider the gravitational back-reaction due to graviton’s self energy on a cosmological background. An explicit two-loop perturbative calculation shows that quantum gravitational effects act to slow the rate of expansion by an amount which becomes non-perturbatively large at late times. The effect is infrared, and is not affected by the ultraviolet difficulties of the theory. Besides being the only two loop calculation in quantum gravity (as far as I know) after the Sagnotti-Goroff non-renormalizability proof, this result is extremely interesting, because, if confirmed, it might represent an effect of quantum gravity with potentially observable consequences

3. Quantum field theory on curved spacetime

Unlike almost anything else described in this report, quantum field theory in curved spacetime is by now a reasonably established theory [64], predicting physical phenomena of remarkable interest such as particle creation, vacuum polarization effects and Hawking’s black-hole radiation [65]. To be sure, there is no direct nor indirect experimental observation of any of these phenomena, but the theory is quite credible as an approximate theory, and many theorists in different fields would probably agree that these predicted phenomena are likely to be real.

The most natural and general formulation of the theory is within the algebraic approach [66], in which the primary objects are the local observables and the states of interest may all be treated on equal footing (as positive linear functionals on the algebra of local observables), even if they do not belong to the same Hilbert space.

In these last years there has been progress in the discussion of phenomena such as the instability of chronology horizons and on the issue of negative energies [67]. Many problems are still open. Interacting fields and renormalization are not yet completely understood, as far as I understand. It is interesting to notice in this regard that the equivalence principle suggests that the problem of the ultraviolet divergences should be of the same nature as in flat space; so no obstruction for renormalization on curved spacetime is visible. Nevertheless, the standard techniques for dealing with the problem are

not viable, mostly for the impossibility of using Fourier decomposition, which is global in nature. If ultraviolet divergences are a local phenomenon, why do we need global Fourier modes to deal with them? A remarkable new development on these issues is the work of Brunetti and Fredenhagen [68]. These authors have found a way to replace the requirement of the positive of energy (which is global) with a novel spectral principle based on the notion of wavefront (which is local). In this way, they make a substantial step towards the construction of a rigorous perturbation theory on curved spaces. The importance of a genuinely local formulation of QFT should probably not be underestimated.

The great merit of QFT on curved spacetime is that it has provided us with some very important lessons. The key lesson is that in general one loses the notion of a single preferred quantum state that could be regarded as *the* “vacuum”; and that the concept of “particle” becomes vague and/or observer-dependent in a gravitational context. These conclusions are extremely solid, and I see no way of avoiding them. In a gravitational context, *vacuum* and *particle* are necessarily ill defined or approximate concepts. It is perhaps regrettable that this important lesson has not been yet absorbed by many scientists working in fundamental theoretical physics.

C. “Unorthodox” approaches

1. Causal sets

Raphael Sorkin vigorously advocates an approach to quantum gravity based on a sum over histories, where the histories are formed by discrete causal sets, or “Posets” [69]. Within this approach, he has discussed black hole entropy [70] and the cosmology constant problem. Sorkin’s ideas have recently influenced various other directions. Markopoulou and Smolin have noticed that one naturally obtains a Poset structure in constructing a Lorentzian version of the spin foam models (see Section V C 2). Connections with noncommutative geometry have been explored in [71] by interpreting the partial ordering as a topology.

2. Finkelstein’s ideas

This year, David Finkelstein, original and radical thinker, has published his book, “Quantum Relativity”, with the latest developments of his profound and fascinating re-thinking of the basis of quantum theory [72]. The book contains a proposal on the possibility of connecting the elementary structure of spacetime with the internal variables (spin, color and isospin) of the elementary particles. The suggestions has resonances with Alain Connes ideas (next Section).

3. Twistors

The twistor program has developed mostly on the classical and mathematical side. Roger Penrose has presented intriguing and very promising steps ahead in this conference [73]. Quantum gravity has been a major motivation for twistors; as far as I know, however, little development has happened on the quantum side of the program.

V. NEW DIRECTIONS

A. Noncommutative geometry

Noncommutative geometry is a research program in mathematics and physics which has recently received wide attention and raised much excitement. The program is based on the idea that spacetime may have a noncommutative structure at the Planck scale. A main driving force of this program is the radical, volcanic and extraordinary sequence of ideas of Alain Connes [74].

Connes’ ideas are many, subtle and fascinating, and I cannot attempt to summarize them all here. I mention only a few, particularly relevant for quantum gravity. Connes observes that what we know about the structure of spacetime derives from our knowledge of the fundamental interactions: special relativity derives from a careful analysis of Maxwell theory; Newtonian spacetime and general relativity derived both from a careful analysis of the gravitational interaction. Recently, we have learned to describe weak and strong interactions in terms of the $SU(3) \times SU(2) \times U(1)$ standard model. Connes suggests that the standard model might hide information on the minute structure of spacetime as well. By making the hypothesis that the standard model symmetries reflect the symmetry of a noncommutative microstructure of spacetime, Connes and Lott are able to construct an exceptionally simple and beautiful version of the standard model itself, with the impressive result that the Higgs field appears automatically, as the components of the Yang Mills connection in the internal “noncommutative” direction [77]. The theory admits a natural extension in which the spacetime metric, or the gravitational field, is dynamical, leading to GR [78].

What is a non-commutative spacetime? The key idea is to use algebra instead of geometry in order to describe spaces. Consider a topological (Hausdorff) space M . Consider all continuous functions f on M . These form an algebra A , because they can be multiplied and summed, and the algebra is commutative. According to a celebrated result, due to Gel’fand, knowledge of the algebra A is equivalent to knowledge of the space M , i.e. M can be reconstructed from A . In particular, the points x of the manifold can be obtained as the (one-dimensional) irreducible representations x of A , which are all of the form $x(f) = f(x)$. Thus, we can use the algebra of the

functions, instead of using the space. In a sense, notices Connes, the algebra is more physical, because we never deal with spacetime: we deal with fields, or coordinates, over spacetime. But one can capture Riemannian geometry as well, algebraically. Consider the Hilbert space H formed by all the spinor fields on a given Riemannian (spin) manifold. Let D be the (curved) Dirac operator, acting on H . We can view A as an algebra of (multiplicative) operators on H . Now, from the triple (H, A, D) , which Connes calls “spectral triple”, one can reconstruct the Riemannian manifold. In particular, it is not difficult to see that the distance between two points x and y can be obtained from these data by

$$d(x, y) = \sup_{\{f \in A, \|Df\| < 1\}} |x(f) - y(f)|, \quad (7)$$

a beautiful surprising algebraic definition of distance. A non-commutative spacetime is the idea of describing spacetime by a spectral triple in which the algebra A is a non-commutative algebra.

Remarkably, the gravitational field is captured, together with the Yang Mills field, and the Higgs fields, by a suitable Dirac operator D [78], and the full action is given simply by the trace of a very simple function of the Dirac operator.

Even if we disregard noncommutativity and the standard model, the above construction represents an intriguing re-formulation of conventional GR, in which the geometry is described by the Dirac operator instead than the metric tensor. This formulation has been explored in [79], where it is noticed that the eigenvalues of the Dirac operator are diffeomorphism invariant functions of the geometry, and therefore represent true observables in (Euclidean) GR. Their Poisson bracket algebra can be explicitly computed in terms of the energy-momentum eigenspinors. Surprisingly, the Einstein equations turn out to be captured by the requirement that the energy momentum of the eigenspinors scale linearly with the eigenvalues.

Variants of Connes’s version of the idea of non commutative geometry and noncommutative coordinates have been explored by many authors [75] and intriguing connections with string theory have been suggested [15,76].

A source of confusion about noncommutative geometry is the use of the expression “quantum”. In the mathematical parlance, one uses the expression “quantization” anytime one replaces a commutative structure with a non-commutative one, whether or not the non-commutativity has anything to do with quantum mechanics.* Models such as the Connes-Lott or the Chamseddine-Connes

models are called “quantum” models by a mathematician, because they are based on a noncommutative algebra, but they are “classical” for a physicist, because they still need to be “quantized”, in order to describe the physics of quantum mechanical phenomena. If the model reduces to a standard Yang-Mills theory, then conventional QFT techniques can be used for the quantization. Thus, for instance, the Connes-Lott models yields the conventional “quantum” standard model. On the other hand, if the model includes a gravitational theory such as GR, which is non-renormalizable, then consistent quantization techniques are missing, and the difficulties of quantum GR are not solved, or mitigated, by just having a noncommutative manifold. In such a model, the replacement of the commutative spacetime manifold with a noncommutative one is not sufficient to address the quantum physics of spacetime.

It is definitely too early to attempt a physical evaluation of the results obtained in this direction. Many difficulties still separate the noncommutative approach from realistic physics. The approach is inspired by Heisenberg intuition that physical observables become noncommutative at a deeper analysis, but so far a true merge with quantum theory is lacking. Even in the classical regime, most of the research is so far in the unphysical Euclidean regime only. (What may replace equation (7) in the Lorentzian case?) Nevertheless, this is a fresh set of new ideas, which should be taken very seriously, and which could lead to crucial advances.

B. Null surface formulation

A second new set of ideas comes from Kozameh, Newman and Frittelli [80]. These authors have discovered that the (conformal) information about the geometry is captured by suitable families of null hypersurfaces in spacetime, and have been able to reformulate GR as a theory of self interacting families of surfaces. Since Carlos Kozameh has described this work in his plenary lecture in this conference [81], I will limit myself to one remark here. A remarkable aspect of the theory is that physical information about the spacetime interior is transferred to null infinity, along null geodesics. Thus, the spacetime interior is described in terms of how we would (literally) “see it” from outside. This description is diffeomorphism invariant, and addresses directly the relational localization characteristic of GR: the spacetime location of a region is determined dynamically by the gravitational field and is captured by when and where we see the spacetime region from infinity. This idea may lead to interesting and physically relevant diffeomorphism invariant observables in quantum gravity. A discussion of the quantum gravitational fuzziness of the spacetime points determined by this perspective can be found in [82].

*Noncommutativity can be completely unrelated to quantum theory, of course. Boosts commute in Galilean relativity and do not commute in special relativity; but this does not mean that special relativity is by itself a quantum theory.

C. Spin foam models

1. Topological quantum field theory

From the mathematical point of view, the problem of quantum gravity is to understand what is QFT on a differentiable manifold without metric (See section VII). A class of well understood QFT's on manifolds exists. These are the topological quantum field theories (TQFT). Topological field theories are particularly simple field theories. They have as many fields as gauges and therefore no local degree of freedom, but only a finite number of global degrees of freedom. An example is GR in 3 dimensions, say on a torus (the theory is equivalent to a Chern Simons theory). In 3d, the Einstein equations require that the geometry is flat, so there are no gravitational waves. Nevertheless, a careful analysis reveals that the radii of the torus are dynamical variables, governed by the theory. Witten has noticed that theories of this kind give rise to interesting quantum models [83], and [84] has provided a beautiful axiomatic definition of a TQFT. Concrete examples of TQFT have been constructed using hamiltonian, combinatorial and path integral methods. The relevance of TQFT for quantum gravity has been suggested by many [84,85] and the recent developments have confirmed these suggestions.

The expression “TQFT” is a bit ambiguous, and this fact has generated a certain confusion. The TQFT's are diffeomorphism invariant QFT. Sometimes, the expression TQFT is used to indicate all diffeomorphism invariant QFT's. This has lead to a widespread, but incorrect belief that any diffeomorphism invariant QFT has a finite number of degrees of freedom, unless the invariance is somehow broken, for instance dynamically. This belief is wrong. The problem of quantum gravity is precisely to define a diffeomorphism invariant QFT having an infinite number degrees of freedom and “local” excitations. Locality in a gravity theory, however, is different from locality in conventional field theory. Let me try to clarify this point, which is often source of confusion:

- In a conventional field theory on a metric space, the degrees of freedom are local in the sense that they can be localized on the metric manifold (an electromagnetic wave is here or there in Minkowski space).
- In a diffeomorphism invariant field theory such as general relativity, the degrees of freedom are still local (gravitational waves exist), but they are not localized with respect to the manifold. They are nevertheless localized with respect to each other (a gravity wave is three meters apart from another gravity wave, or from a black hole).
- In a topological field theory, the degrees of freedom are not localized at all: they are global, and in finite number (the radius of a torus is not in a particular position on the torus).

Let me illustrate the main steps of the winding story. The first TQFT directly related to quantum gravity was defined by Turaev and Viro [96]. The Turaev-Viro model is a mathematically rigorous version of the 3d Ponzano-Regge quantum gravity model described in section IV A 3. In the Turaev-Viro theory, the sum (5) is made finite by replacing $SU(2)$ with quantum $SU(2)_q$ (with a suitable q). Since $SU(2)_q$ has a finite number of irreducible representations, this trick, suggested by Ooguri, makes the sum finite. The extension of this model to four dimensions has been actively searched for a while and has finally been constructed by Louis Crane and David Yetter, again following Ooguri's ideas [86,87]. The Crane-Yetter (CY) model is the first example of 4d TQFT. It is defined on a simplicial decomposition of the manifold. The variables are spins (“colors”) attached to faces and tetrahedra of the simplicial complex. Each 4-simplex contains 10 faces and 5 tetrahedra, and therefore there are 15 spins associated to it. The action is defined in terms of the (quantum) Wigner 15- j symbols, in the same manner in which the Ponzano-Regge action is constructed in terms of products of 6- j symbols.

$$Z \sim \sum_{\text{coloring}} \prod_{4\text{-simplices}} 15\text{-}j(\text{color of the 4-simplex}), \quad (8)$$

(I disregard various factors for simplicity). Crane and Yetter introduced their model independently from loop quantum gravity. However, recall from Section IV A 3 that loop quantum gravity suggests that in 4 dimensions the naturally discrete geometrical quantities are area and volume, and that it is natural to extend the Ponzano-Regge model to 4d by assigning colors to faces and tetrahedra.

The CY model is not a quantization of 4d GR, nor could it be, being a TQFT in strict sense. Rather, it can be formally derived as a quantization of $SU(2)$ BF theory. BF theory is a topological field theory with two fields, a connection A , with curvature F , and a two-form B [88], with action

$$S[A, B] = \int B \wedge F. \quad (9)$$

However, there is a strict relation between GR and BF. If we add to $SO(3, 1)$ BF theory the constraint that the two-form B is the product of two tetrad one-forms

$$B = E \wedge E, \quad (10)$$

we obtain precisely GR [89,90]. This observation has lead many to suggest that a quantum theory of gravity could be constructed by a suitable modification of quantum BF theory [91]. The suggestion has recently become very plausible, with the construction of the spin foam models, described below.

The key step was taken by Andrea Barbieri, studying the “quantum geometry” of the simplices that play a role in loop quantum gravity [93]. Barbieri discovered a simple relation between the quantum operators representing the areas of the faces of the tetrahedra. This relation turns out to be the quantum version of the constraint (10), which turns BF theory into GR. Barret and Crane [94] added the Barbieri relation to (the $SO(3,1)$ version of) the CY model. This is equivalent to replacing the the 15- j Wigner symbol, with a different function A_{BC} of the colors of the 4-simplex. This replacement defines a “modified TQFT”, which has a chance of having general relativity as its classical limit. Details and an introduction to the subject can be found in [95].

The Barret-Crane model is not a TQFT in strict sense. In particular, it is not independent from the triangulation. Thus, a continuum theory has to be formally defined by some suitable sum over triangulations

$$Z \sim \sum_{\text{triang}} \sum_{\text{coloring}} \prod_{4\text{-simplices}} A_{BC}(\text{color of the 4-simplex}). \quad (11)$$

This essential aspect of the construction, however, is not yet understood.

The striking fact is that the Barret Crane model can virtually be obtained also from loop quantum gravity. This is an unexpected convergence of two very different lines of research. Loop quantum gravity is formulated canonically in the frozen time formalism. While the frozen time formalism is in principle complete, in practice it is cumbersome, and anti-intuitive. Our intuition is four dimensional, not three dimensional. An old problem in loop quantum gravity has been to derive a spacetime version of the theory. A spacetime formulation of quantum mechanics is provided by the sum over histories. A sum over histories can be derived from the hamiltonian formalism, as Feynman did originally. Loop quantum gravity provides a mathematically well defined hamiltonian formalism, and one can therefore follow Feynman steps and construct a sum over histories quantum gravity starting from the loop formalism. This has been done in [92]. The sum over histories turns out to have the form of a sum over surfaces.

More precisely, the transition amplitude between two spin network states turns out to be given by a sum of terms, where each term can be represented by a (2d) branched “colored” surface in spacetime. A branched colored surface is formed by elementary surface elements carrying a label, that meet on edges, also carrying a label; edges, in turn meet in vertices (or branching points). See Figure 3

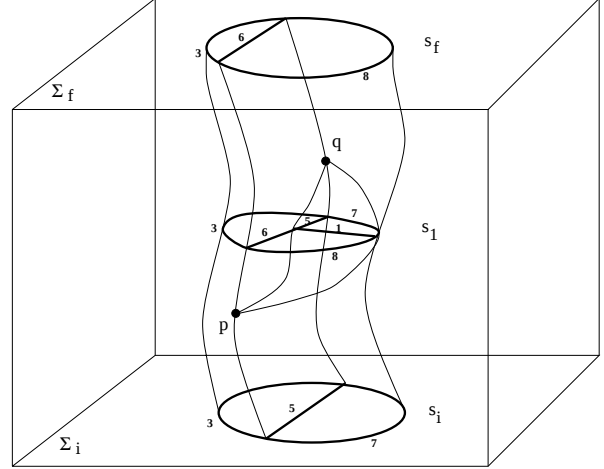


FIG. 3. Figure 3: A branched surface with two vertices.

The contribution of one such surfaces to the sum over histories is the product of one term per each branching point of the surface. The branching points represent the “vertices” of this theory, in the sense of Feynman. See Figure 4.

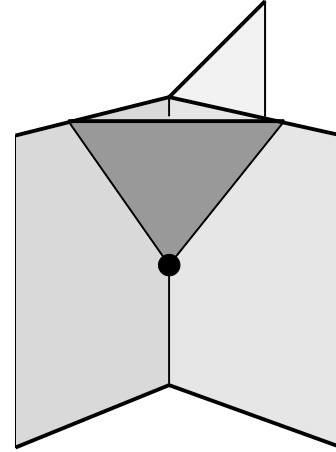


FIG. 4. Figure 4: A simple vertex.

The contribution of each vertex can be computed algebraically from the “colors” (half integers) of the adjacent surface elements and edges. Thus, spacetime loop quantum gravity is defined by the partition function

$$Z \sim \sum_{\text{surfaces}} \sum_{\text{colorings}} \prod_{\text{vertices}} A_{\text{loop}}(\text{color of the vertex}) \quad (12)$$

The vertex A_{loop} is determined by a matrix elements of the hamiltonian constraint. The fact that one obtains a

sum over surfaces is not too surprising, since the time evolution of a loop is a surface. Indeed, this was conjectured time ago by Baez and by Reisenberger. The time evolution of a spin network (with colors on links and nodes) is a surface (with colors on surface elements and edges) and the hamiltonian constraint generates branching points in the same manner in which conventional hamiltonians generate the vertices of the Feynman diagrams.

What *is* surprising is that (12) has the same structure of the Barret Crane model (8). To see this, simply notice that we can view each branched colored surface as located on the lattice *dual* to a triangulation (recall Figure 2). Then each vertex correspond to a 4-simplex; the coloring of the two models matches exactly (elementary surfaces $\rightarrow$ faces, edges $\rightarrow$ tetrahedra); and summing over surfaces corresponds to summing over triangulations. The main difference is the different weight at the vertices. The Barret-Crane vertex A_{BC} can be read as a covariant definition a hamiltonian constraint in loop quantum gravity.

Thus, the spacetime formulation of loop quantum GR is a simple modification of a TQFT. This approach provides a 4d pictorial intuition of quantum spacetime, analogous to the Feynman graphs description of quantum field dynamics. John Baez has introduced the term “spin foam” for the branched colored surfaces of the model, in honor of John Wheeler’s intuitions on the quantum microstructure of spacetime. Spin foams are a precise mathematical implementation of Wheeler’s “spacetime foam” suggestions. Markopoulou and Smolin have explored the Lorentzian version of the spin foam models [97].

This direction is very recent. It is certainly far too early to attempt an evaluation. Many aspects of these models are still obscure. But the spacetime foam models may turn out among the most promising recent development in quantum gravity.

This concludes the survey of the main approaches to quantum gravity.

VI. BLACK HOLE ENTROPY

A focal point of the research in quantum gravity in the last years has been the discussion of black hole (BH) entropy. This problem has been discussed from a large variety of perspectives and within many different research programs.

Let me very briefly recall the origin of the problem. In *classical* GR, future event horizons behave in a manner that has a peculiar thermodynamical flavor. This remark, together with a detailed physical analysis of the behavior of hot matter in the vicinity of horizons, prompted Bekenstein, over 20 years ago, to suggest that there is entropy associated to every horizon. The suggestion was

first consider ridicule, because it implies that a black hole is hot and radiates. But then Steven Hawking, in a celebrated work [65], showed that QFT in curved spacetime predicts that a black hole emits thermal radiation, precisely at the temperature predicted by Bekenstein, and Bekenstein courageous suggestion was fully vindicated. Since then, the entropy of a BH has been indirectly computed in a surprising variety of manners, to the point that BH entropy and BH radiance are now considered almost an established fact by the community, although, of course, they were never observed nor, presumably, they are going to be observed soon. This confidence, perhaps a bit surprising to outsiders, is related to the fact thermodynamics is powerful in indicating general properties of systems, even if we do not control its microphysics. Many hope that the Bekenstein-Hawking radiation could play for quantum gravity a role analogous to the role played by the black body radiation for quantum mechanics.

Thus, indirect arguments indicate that a Schwarzschild BH has an entropy

$$S = \frac{1}{4} \frac{A}{\hbar G} \quad (13)$$

The challenge is to derive this formula from first principles. A surprisingly large number of derivations of this formula have appeared in the last years.

String theory. In string theory, one can count the number of string states that have the same mass and the same charges at infinity as an extremal BH. (An extremal BH is a BH with as much charge as possible – astrophysically, an extremal BH is a highly improbable object). Since a BH is a nonperturbative object, the calculation refers to the nonperturbative regime, where string theory is poorly understood. But it can nevertheless be completed, thanks to a trick. At fixed mass and charge at infinity, if the coupling constant is large, there is a BH, but we do not control the theory; if the coupling is weak there is no BH, but the theory is in the perturbative, and we can count states with given mass and charges. Thanks to a (super-) symmetry of string theory, quantum states corresponding to extremal BH (BPS states) have the property that their number does not depend on the strength of the coupling constant. Therefore we can count them in the limit of weak coupling, and be confident that the counting holds at strong coupling as well. In this way we can compute how many states in the theory (at strong coupling) correspond to a black hole geometry [8,9]. The striking results is that if we interpret BH entropy as generated by the number of such states ($S = k \ln N$) we obtain the correct Bekenstein Hawking formula, with the correct $1/4$ factor.

The derivation has been extended outside the extremal case [10], but I am not aware, so far, of a result for non extremal BH’s as clean and compelling

as the result for the extremal case. The Hawking radiation rate itself can be derived from the string picture (for the near extremal black holes) [11]. For long wavelength radiation, one can also calculate the ‘grey body factor’, namely corrections to the thermal spectrum due to frequency-dependent potential barriers outside the horizon, which filter the initially blackbody spectrum emanating from the horizon [12].

The derivation is striking, but it leaves many open questions. Are the distinct BH states that enter the counting distinguishable from each other, for an observer at infinity? If they are not distinguishable, how can they give rise to thermodynamical behavior? (Entropy is the number of *distinguishable* states.) If yes, which observable can distinguish them? Is a black hole in string theory not really black? (See [13].) More in general, why is there this consistency between string theory, QFT in curved space and classical GR? How does thermodynamics, horizons and quantum theory interplay?

Of course, one cannot view the BH entropy derivation as experimental support for string theory: BH radiance has never been observed, and, even if it is observed, BH radiance was predicted by Hawking, not by string theory, and it is just a consequence of QFT on a curved background plus classical GR, not of string theory. Any theory of quantum gravity consistent with the QFT in curved spacetime limit should yield the BH entropy. What the string derivation does show is that string theory is indeed consistent with GR and with QFT on curved space, even in the strong field regime, where the theory is poorly understood, and at least as far as the extremal case is concerned. The derivation of the entropy formula for the extremal BH represents a definite success of string theory.

Surface states. Fifteen years ago, York suggested that the degrees of freedom associated to BH entropy could be interpreted as fluctuations of the position of the event horizon [98]. Thus, they could reside on the horizon itself. This suggestion has recently become precise. The new idea is that the horizon (in a precise sense) breaks diffeomorphism invariance locally, and this fact generates quantum states on the BH surface [99–101]. These states are called edge states, or surface states, and can naturally be described in terms of a topological theory on the horizon. Balachandran, Chandar and Momen have derived the existence of surface states in 3+1 gravity, and showed that these are described by a surface TQFT [100]. Steven Carlip has shown that this idea leads to a computation of the BH entropy in 2+1 gravity, obtaining the correct $1/4$ factor [99]. The surface states idea is also at the root of the

loop quantum gravity derivations described below.

Loop Quantum Gravity. Kirill Krasnov has introduced statistical techniques for counting loop microstates [102] and has opened the study of BH entropy within loop quantum gravity. There are two derivations of BH entropy from loop quantum gravity. One [103,102] is based on a semiclassical analysis of the physics of a hot black hole. This analysis suggests that the area of the horizon does not change while it is thermally “shaking”. This implies that the thermal properties of the BH are governed by the number of microstates of the horizon having the same area. The apparatus of loop quantum gravity is then employed to compute this number, which turns out to be finite, because of the Planck scale discreteness implied by the existence of the quanta of geometry (Section III B 1). The number of relevant states is essentially obtained from the number of the eigenvalues in equation (3) that have a given area. In the second approach [104], one analyzes the classical theory *outside* the horizon treating the horizon as a boundary. A suitable quantization of this theory yields surface states, which turn out to be counted by an effective Chern Simons theory on the boundary, thus recovering the ideas of Balachandran and collaborators. In both derivations, one obtains, using Eq.(3), that the entropy is proportional to the area in Planck units. However, loop quantum gravity does not fix the (finite) constant of proportionality, because of the parameter γ in (3), a finite free dimensionless parameter not determined by the theory, first noticed by Immirzi [105,106].

In comparison with the string derivation, the loop derivation is weaker because it does not determine univocally the $1/4$ factor of Eq.(13) and it is stronger because it works naturally for “realistic” BH’s, such as Schwarzschild.

Entanglement entropy. An old idea about BH entropy, first considered by Bombelli, Koul, J Lee and Sorkin is that it is the effect of the short scale quantum entanglement between the two sides of the horizon [107]. A similar idea was independently proposed by Frolov and Novikov, who suggest that BH entropy reflects the degeneracy with respect to different quantum states which exist inside the black hole, where inside modes contribute only if they are correlated with external modes [108]. The idea of entanglement entropy has been recently analyzed in detail in [109], where it is suggested that, suitably interpreted, the idea might still be valid.

Induced gravity. Frolov and Fursaev have developed the idea of entanglement entropy by applying it within Sakharov’s induced gravity theories, following a suggestion by Ted Jacobson. The idea of using

induced gravity theories is motivated by the fact that the bare gravitational constant gets renormalized in the computation of the entanglement entropy, yielding a divergent entropy [110]. In induced gravity theories, there is no bare gravitational constant, and one may obtain the correct finite answer. The idea is that a (still unknown) fundamental theory which induces the correct low energy gravity should allow a representation in terms of an infinite set of fields which could play the role of the induced gravity constituents.

Bekenstein’s model. Bekenstein, the “inventor” of BH entropy, has recently analyzed the quantum structure of BH’s, using the idea that a BH can be treated as simple quantum objects. This quantum object has, presumably, quantized energy levels, or horizon-area levels, since energy and area are related for a BH. Therefore it emits energy in quantum jumps, as atoms do. An interesting consequence of this approach [111] is that if the horizon area levels are equispaced, the emitted radiation differs strongly from the thermal one predicted by Hawking, and presents macroscopically spaced spectral lines. This would be extremely interesting, because these spectral lines might represent observable macroscopic QG effects. Unfortunately, several more complete approaches, and in particular, loop quantum gravity, do not lead to equispaced area levels (see Eq.(3)). With a more complicated area spectrum the emitted radiation is effectively thermal [112,38], as predicted by Hawking. Nevertheless, the Bekenstein-Mukhanov effect remains an intriguing idea: for instance, it has been suggested that it might actually resurrect in loop quantum gravity for dynamical reasons.

’t Hooft’s “S-matrix ansatz” and “holographic principle”. In conjunction with his discussion of BH’s radiation, Steven Hawking has long claimed that BH’s violate ordinary quantum mechanics, in the sense that a pure state can evolve into a mixed state in the presence of a BH. More precisely, the evolution from $t = -\infty$ to $t = +\infty$ is not given by the S -matrix acting on physical states, but rather by an operator, which he calls $\mathcal{S}$ -matrix, acting on density matrices. Gerard ’t Hooft has been disputing this view for a long time, maintaining that the evolution should still be given by an S -matrix. ’t Hooft observes that if one assumes the validity of conventional quantum field theory in the vicinity of the horizon, one does not find a quantum mechanical description of the BH that resembles that of conventional forms of matter. Instead, he considers the alternative assumption that a BH can be described as an ordinary object within unitary quantum theory. The assumption of the existence of an ordinary S -matrix has far reaching conse-

quences on the nature of space-time, and even on the description of the degrees of freedom in ordinary flat space-time. In particular, the fact that all microstates are located on the horizon implies a puzzling property of space-time itself, denoted the holographic principle. According to this principle, the combination of quantum mechanics and gravity requires the three dimensional world to be an image of data that can be stored on a two dimensional projection – much like a holographic image. The two dimensional description only requires one discrete degree of freedom per Planck area and yet it is rich enough to describe all three dimensional phenomena. These views have recently been summarized in Ref. [113]. Susskind has explored some consequences of ’t Hooft’s holographic principle, showing that it implies that particles must grow in size as their momenta increase far above the Planck scale, a phenomenon previously discussed in the context of string theory, thus opening a possible connection between ’t Hooft views and string theory [114].

Trans-Planckian frequencies. Work by Unruh and Jacobson has provided interesting insight into how the prediction of Hawking radiation apparently is not affected by modifications of the theory at ultra-high frequencies. If the modes of the Hawking radiance are red shifted emerging from a BH, one might imagine that finite frequencies at infinity derive from arbitrary high frequencies at the horizon. But if spacetime is discrete, arbitrary high modes do not exist. One can get out from this apparent paradox by observing that the outgoing modes do not arise from high frequency modes at the horizon, but from ingoing modes, through a process of “mode conversion” which is well known in plasma physics and in condensed matter physics [115].

Others. Several others results on black hole entropy exist [70]. I do not have the space or the competence for an exhaustive list. For a recent overview, see Ted Jacobson review of the BH entropy section of the MG8 [116].

The above list shows that there is a rather large number of research programs on BH entropy. Many of these programs claim that a key for solving the puzzle has been found. However most research program ignore the others. Presumably, what is required now is a detailed comparison of the various ideas.

VII. THE PROBLEM OF QUANTUM GRAVITY. A DISCUSSION

The problem that the research on quantum gravity addresses is simply formulated: finding a fundamental theoretical description of the physics of the gravitational field

in the regime in which its quantum mechanical properties cannot be disregarded.

This problem, however, is interpreted in surprisingly different manners by physicists with different cultural backgrounds. There are two main interpretations of this problem, driving the present research: the particle physicist's one and the relativist's one.

1. The problem, as seen by a high energy physicist

High energy physics has obtained spectacular successes during this century, culminated with the establishment of quantum field theory and of the $SU(3) \times SU(2) \times U(1)$ standard model. The standard model encompasses virtually everything we can physically measure – except gravitational phenomena. From the point of view of a particle physicist, gravity is simply the last and weakest of the interactions. It is natural to try to understand its quantum properties using the same strategy that has been so successful for the rest of microphysics, or variants of this strategy. The search for a conventional quantum field theory capable of embracing gravity has spanned several decades and, through a curious sequence of twists, excitements and disappointments, has lead to string theory.

For a physicist with a high energy background, the problem of quantum gravity is thus reduced to an aspect of the problem of understanding what is the mysterious nonperturbative theory that has perturbative string theory as its perturbation expansion, and how to extract information on Planck scale physics from it.

In string theory, gravity is just one of the excitations of a string (or other extended object) living over some metric space. The existence of such background metric space, over which the theory is defined, is needed for the formulation of the theory, not just in perturbative string theory, but also in most of the recent attempts of a non-perturbative definition of the theory, as I argued in section III A.

2. The problem, as seen by a relativist

For a relativist the idea of a fundamental description of gravity in terms of physical excitations over a metric space sounds incorrect. The key lesson of GR is that there is no background metric over which physics happens (except in approximations). The gravitational field is the same physical object as the spacetime itself, and therefore quantum gravity is the theory of the quantum microstructure of spacetime. To understand quantum gravity we have to understand what is quantum spacetime.

More precisely, for a relativist, GR is much more than the field theory of a particular force. Rather, it is the discovery that certain classical notions about space and time are not adequate at the fundamental level; and require

a deep modifications. One of such inadequate notions is precisely the notion of a background metric space (flat or curved), *over* which physics happens. It is this conceptual shift that has led to the understanding of relativistic gravity, to the discovery of black holes, to relativistic astrophysics and to modern cosmology. For a relativist, quantum gravity is the problem of merging *this* conceptual novelty with quantum field theory.

From Newton to the beginning of this century, physics has been founded over a small number of key notions such as space, time, causality and matter. In the first quarter of this century, quantum theory and general relativity have modified this foundation in depth. The two theories have obtained solid success and vast experimental corroboration, and can be now considered as well established knowledge. Each of the two theories modifies the conceptual foundation of classical physics in a (more or less) internally consistent manner. However, we do not have a novel conceptualization of the physical world capable of embracing both theories. For a relativist, the challenge of quantum gravity is the problem of bringing this vast conceptual revolution, started with quantum mechanics and with general relativity, to a conclusion and to a new synthesis.

Unlike perturbative or nonperturbative string theory, relativist's quantum gravity theories tend to be formulated without a background spacetime, and are direct attempts to grasp what is quantum spacetime at the fundamental level.

3. What is quantum spacetime?

General relativity has taught us that the spacetime metric is dynamical, like the rest of the physical entities. From quantum mechanics we have learned that all dynamical entities have quantum properties (undergo quantum fluctuations, are quantized, namely they tend to manifest themselves in small quanta at short scale, and so on). These quantum properties are captured by the basic formalism of quantum mechanics, in its various versions. Thus, we expect spacetime metric to be subject to Heisenberg's uncertainty principle, to come in small packet, or quanta of spacetime, and so on. Spacetime metric should then only exist as an expectation value of some quantum variable.

But we have learned another more general lesson from GR: that spacetime location is relational only. This is a distinct idea from the fact that the metric is dynamical. Mathematically, this physical idea is captured by the active active diff invariance of the Einstein equations. (Einstein searched for *non-diff invariant* equations for a dynamical metric and for the Riemann tensor from 1912 to 1915, before understanding the need of active diff invariance in the theory.) Active diff invariance means that the theory is invariant under a diffeomorphism on the dynamical fields of the theory (not on every object of the

theory: any theory, suitably formulated is trivially invariant under a diffeomorphism on all its objects). Physically, diff invariance has a profound and far reaching meaning. This meaning is subtle, and even today, 75 years after the discovery of GR, it is sometimes missed by theoretical physicists, particularly physicists without a GR background.

A non diff-invariant theory of a system S describes the evolution of the objects in S with respect to a reference system made by objects external to S . A diff-invariant theory of a system S describes the dynamics of the objects in S with respect to each others. In particular, localization is defined only internally, relationally. Objects are somewhere only with respects to other dynamical objects of the theory, not with respect to an external reference system. The electromagnetic field of Maxwell theory is located somewhere in spacetime. The gravitational field is not located in spacetime: it is with respect to *it* that things are localized. To put it pictorially, pre-GR physics describes the motion of physical entities over the stage formed by a non-dynamical spacetime. While general relativistic physics describes the dynamics of the stage itself. The stage does not “move” over a background. It “moves” with respects to itself. Therefore, what we need in quantum gravity is *a relational notion of a quantum spacetime*.

General quantum theory does not seem to contain any element incompatible with this physical picture. On the other hand, conventional quantum *field* theory does, because it is formulated as a theory of the motion of small excitations over a background. Thus, to merge general relativity and quantum mechanics we need a quantum theory for a field system, but different from conventional QFT over a given metric space. General relativity, as a classical field theory, is not defined over a metric space, but over a space with a much weaker structure: a differentiable manifold. Similarly, in quantum gravity we presumably need a QFT that lives over a manifold. Mathematically, the challenge of quantum gravity can therefore be seen as the challenge of understanding how to consistently define a QFT over a manifold, as opposite to a QFT over a metric space. The theory must respects the manifold invariance, namely active diffeomorphism. This means that the location of states on the manifold is irrelevant.

This idea was beautifully expressed by Roger Penrose in the work in which he introduced spin networks [118].

“A reformulation is suggested in which quantities normally requiring continuous coordinates for their description are eliminated from primary consideration. In particular, since space and time have therefore to be eliminated, what might be called a form of Mach’s principle be invoked: a relationship of an object to some background space should not be considered – only relationships of objects to each other can have significance.”

Several of the research programs described above realize this program to a smaller or larger extent. In particular, recall that the spin network states of loop quantum gravity (See Figure 1) are not excitations *over* spacetime. They are excitations *of* spacetime. This relational aspect of quantum gravitational states is one of the most intriguing aspects emerging from the theory.

4. Quantum spacetime, other aspects

The old idea of a lower bound of the divisibility of space around the Planck scale has been strongly reinforced in the last years. Loop quantum gravity has provided *quantitative* evidence in this sense, thanks to the computation of the quanta of area and volume. The same idea appears in string theory, in certain aspects of non commutative geometry, in Sorkin’s poset theory, and in other approaches [71].

Notice, in this regard, that spacetime is discrete in the quantum sense. It is not “made by discrete quanta”, in the sense in which an electron is not “made by Bohr orbitals”. A generic spacetime is a quantum superposition of discretized states. Outcome of measurements can be discrete, expectation values are continuous.

Physically, one can view the Planck scale discreteness as produced by short scale quantum fluctuations: at the scale at which these are sufficiently strong, the virtual energy density is sufficient to produce micro black holes. In other words, flat spacetime is unstable at short scale. A recent variational computation [125] confirms this idea by showing that flat spacetime has higher energy than a spacetime made of Planck scale black holes.

Another old idea that has consequently been reinforced in the last years is that the perturbative picture of a Minkowski space with real and virtual gravitons is not appropriate at the Planck scale. For instance, the string black hole computation does not works because the weak coupling expansion reaches the relevant regime, but because there is a special case in which one can independently argue that the number of states is the same at weak and strong coupling.

In general, thus, there seem to be a certain convergence in the emerging physical picture of Planck scale quantum spacetime. However, we are far from a point in which we can say that we understand the structure of quantum spacetime, and many general questions remain open. In loop quantum gravity, a credible state representing Minkowski has not been found yet. In string theory, there are too many vacua and the theory does not seem to have much predictivity about the details of the Planck scale structure of spacetime. In the discrete approach, it is not yet clear whether the phase transition gives rise to a large scale theory, and, if so, whether the discrete structure of the triangulations leaves a physical remnant (in QCD it does not, of course). A general problem is the precise relation between spacetime’s mi-

crophysics and macrophysics. Do we expect a full fledged renormalization group to play a role? Do we expect large scale physics to be insensitive to the details of the microphysics, as happens in renormalizable QFT? Or the existence of a physical cutoff kills this idea? Smolin, has suggested that the existence of a phase transition should not be a defining property of the theory, but rather a property of certain states in the theory, the ones that yield macroscopic spacetimes instead of Planck scale clots. Much is still unclear about quantum spacetime.

VIII. RELATION BETWEEN QUANTUM GRAVITY AND OTHER MAJOR OPEN PROBLEMS IN FUNDAMENTAL PHYSICS

When one contemplates two deep problems, one is immediately tempted to speculate that they are related. Quantum gravity has been asked, at some time or the other, to take charge of almost every other open problem in theoretical physics (and beyond). Here is a list of problems that at some time or another have been connected to quantum gravity.

It is important to remark that, with few important exceptions, these problems might very well turn out to be unrelated to quantum gravity. The history of physics is full of examples of two problems solved together (say: understanding the nature of light and uniting electricity to magnetism). But it is also full of disappointed great hopes of getting two results with one stroke (say: finding a theory of the strong interactions and getting rid of ultraviolet divergences and infinite renormalization). In particular, the fact that a proposed solution to the quantum gravity puzzle does not address this or that of the following problems is definitely not an indication it is physically wrong. QCD was initially criticized as a theory of strong interactions because it did not solve the puzzles raised by renormalization theory. We should not repeat that mistake.

I begin with various issues related to quantum mechanics, which are sometimes confused with each other.

Quantum Cosmology. There is widespread confusion between quantum cosmology and quantum gravity. Quantum cosmology is the theory of the entire universe as a quantum system without external observer [119,120]. The problem of quantum cosmology exists with or without gravity. Quantum gravity is the theory of one dynamical entity: the quantum gravitational field (or the spacetime metric): just one entity among the many. We can assume that we have a classical observer with a classical measuring apparatus measuring quantum gravitational phenomena, and therefore we can formulate quantum gravity disregarding quantum cosmology. In particular, the physics of a Planck size small cube is governed by quantum gravity and, presumably, has no cosmological implications. Quantum

cosmology addresses an extremely general and important open question. But that question is not necessarily tied to quantum gravity.

Quantum theory “without time”. Unitarity.

The relational character of GR described in Section VII 3 is reflected in the peculiar role of time in gravity. GR does not describe evolution with respect to an external time, but only relative evolution of physical variables with respect to each other. In other words, temporal localization is relational like spatial localization. This is reflected in the fact that the theory has no hamiltonian (unless particular structures are added), but only a “hamiltonian” constraint. Conventional quantum mechanics needs to be adapted to this way of treating time. There are several ways of doing so. Sum over histories may be a particularly suitable way of formulating such “generalized” quantum mechanics in a gravitational context, as suggested by the work of Jim Hartle [120]; canonical methods are viable as well [123]. For an extensive discussion of the problem and its many subtleties, see [124].

Opinions diverge on whether a definition of time evolution must be unitary in nonperturbative quantum gravity. If we assume asymptotic flatness, then there is a preferred time at infinity and Poincaré symmetry at infinity implies unitarity. Outside this case, the issue is much more delicate. Unitarity is needed for the consistency of a theory in flat space. But the requirement of unitarity should probably not be mistaken for a general consistency requirement, and erroneously extended from the flat space domain, where there is an external time, to the quantum gravity domain, where there is no external time. In GR, one can describe evolution with respect to a rather arbitrarily chosen physical time variable T . There is no reason for a T -dependent operator $A(T)$ to be unitarily related to $A(0)$. Lack of unitarity simply means that the time evolution of a complete set of commuting observables may fail to be a complete set of commuting observables. This is an obstruction for the definition of a Schrödinger picture of time evolution, but the Heisenberg picture [123], or the path integral formulation [120], may nevertheless be consistent.

Structure and interpretation of quantum mechanics. Topos theory. It has been often suggested that the much debated interpretative difficulties of quantum theory may be related to quantum gravity, or that the very structure of quantum mechanics might have to undergo a substantial revision in order to include GR. In Ted Newman’s views, for instance, the gravitational field is so physically different from any other field, that conventional quantizations methods,

“another form of orthodoxy”,

as Ted calls them, are unlikely to succeed. Thus, Newman advocates the need of a substantial revision of quantum theory in order to understand quantum gravity, and expects that the mysteries of quantum gravity and the mysteries of quantum mechanics be intertwined.

Certainly, quantum gravity and quantum cosmology have played an indirect role in the effort to understand quantum theory. If quantum theory has to play the role of general theory of mechanics, it certainly has to be general enough to encompass the peculiar features of gravitational theories as well. In particular, the consistent-histories approach to quantum theory [126] was motivated in part by the search for an interpretation viable in a context where the microstructure of spacetime itself is subject of quantum effects.

A fascinating development in this direction is the recent work of Chris Isham on the relevance of topos-theory in the histories formulation of quantum mechanics [127]. The main idea is to assign to each proposition P a truth value defined as the set (the “sieve”) of all consistent families of histories within which P holds. The set of all such sieves forms a logical algebra, albeit one that contains more than just the values ‘true’ and ‘false’. This algebra is naturally described by topos theory. Isham’s topos-theoretical formulation of quantum mechanics is motivated in part by the desire of extending quantum theory to contexts in which a classical spacetime does not exist. More in general, topos theory has a strongly relational flavor and emphasizes relational aspects of quantum theory. (Relational aspects of quantum theory are discussed also in [128].) The existence of a connection between such relational aspects of quantum theory and relational aspects of GR (Section VII 3) has been explicitly suggested in Refs. [130,129], and might represent a window over a still unexplored realm. These difficult issues are still very poorly understood, but they could turn out to be crucial for future developments.

Wave function collapse. A direct implementation of the idea that the mysteries of quantum gravity and the mysteries of quantum mechanics can be related is Penrose’s suggestion that the wave function collapse may be a gravitational phenomenon. Penrose’s idea is that there may be a nonlinear dynamical mechanism that forbids quantum superpositions of (“too different”) spacetimes. A fact that perhaps supports the speculation is the disconcerting value of the Planck mass. The Planck mass, 22 micrograms, lies approximately at the boundary between the light objects that we see behaving mostly quantum mechanically and the heavy objects that we see behaving mostly classically. Since

the Planck mass contains the Newton constant, this coincidence might be read as an indication that gravity plays a role in a hypothetical transition between quantum and classical physics. Consider an extended body with mass M in a quantum superposition of two states Ψ_1 and Ψ_2 in which the center of mass is, respectively, in the positions X_1 and X_2 . Let U_{grav} be the gravitational potential energy that two distinct such bodies would have if they were in X_1 and X_2 . Penrose suggests that the quantum superposition $\Psi_1 + \Psi_2$ is unstable and naturally decays through some not yet known dynamics to either Ψ_1 or Ψ_2 , with a decay time

$$t_{collapse} \sim \frac{\hbar}{U_{grav}}. \quad (14)$$

The decay time (14) turns out to be surprisingly realistic, as one can easily compute: a proton can be in a quantum superposition for eons, a drop of water decays extremely fast, and the transition region in which the decay time is of the order of seconds is precisely in the regime in which we encounter the boundary between classical and quantum behavior.

The most interesting aspect of Penrose’s idea is that it can be tested in principle, and perhaps even in practice. Antony Zeilinger has announced in his plenary talk in this conference [122] that he will try to test this prediction in the laboratory. Most physicists would probably expect that conventional quantum mechanics will once more turn out to be exactly followed by nature, and the formula (14) will be disproved. But it is certainly worthwhile checking.

Unifications of all interactions and “Theory of Everything”. String theory represents a tentative solution of the quantum gravity problem, but also of the problem of unifying all presently known fundamental physics. This is a fascinating and attractive aspect of string theory. On the other hand, this is not a reason for discarding alternatives. The idea that quantum gravity can be understood only in conjunctions with other matter fields is an interesting hypothesis, not an established truth.

Origin of the Universe. It is likely that a sound quantum theory of gravity will be needed to understand the physics of the Big Bang. The converse is probably not true: we should be able to understand the small scale structure of spacetime even if we do not yet understand the origin of the Universe.

Ultraviolet divergences. As already mentioned, a great hope during the search for the fundamental theory of the strong interactions was to get rid of the QFT’s ultraviolet divergences and infinite renormalization. The hope was disappointed,

but QCD was found nevertheless. A similar hope is alive for quantum gravity, but this time the perspectives look better. Perturbative string theory is (almost certainly) finite order by order, and loop quantum gravity reveals a discrete structure of space at the Planck scale which, literally, “leaves no space” for the ultraviolet divergences.

IX. CONCLUSION

We have at least two well developed, although still incomplete, theories of quantum spacetime, string theory and loop quantum gravity. Both theories provide a physical picture and some detailed results on Planck scale physics. The two pictures are quite different, in part reflecting the diverse cultures from which they originated, high energy physics and relativity. In addition, a number of promising fresh ideas and fresh approaches have recently appeared, most notably noncommutative geometry. The main physical results on quantum spacetime obtained in the last three years within these theories are the following.

- A striking result is the explicit computation of the quanta of geometry, namely the discrete spectra of area and volume, obtained in loop quantum gravity.
- Substantial progress in understanding black hole entropy has been achieved in string theory, in loop quantum gravity, and using other techniques.
- Two cosmological applications of quantum gravity have been proposed. String cosmology might yield predictions on the spectrum of the background gravitational radiation. According to Woodard and Tasmis, two-loops quantum gravity effects might be relevant in some cosmological models.

Among the most serious open problems are the following.

- Black hole entropy has been discussed using a variety of different approaches, and the relation between the various ideas is unclear. What is needed in black hole thermodynamics is a critical comparison between the many existing ideas about the source of BH entropy, and possibly a synthesis.
- In string theory, the key problem in view of the description of quantum spacetime is to find the background-independent formulation of the theory.
- In loop quantum gravity, the main problem is to understand the low energy limit and to single out the correct version of the hamiltonian constraint. A promising direction in this regard might be given by the spin foam models.

- In noncommutative geometry, the problem that probably needs to be understood is the relation between the noncommutative structure of spacetime and the quantum field theoretical aspects of the theory. In particular, how is renormalization affected by the spacetime noncommutativity?

The relations between various approaches may be closer than expected. I have already pointed out some noncommutative geometry aspects of string theory. String theory and loop quantum gravity are remarkably complementary in their successes, and one may speculate that they could merge or that some technique could be transferred from one to the other. In particular, loop quantum gravity is successful in dealing with the nonperturbative background-independent description of quantum spacetime, which is precisely what is missing in string theory, and loops might provide some tools to strings. A loop, of course, is not a very different object from a string. String theory can be formulated as a sum over world-sheets, and loop quantum gravity can be formulated as a sum over surfaces. The world sheets of string theory do not branch and are defined over a metric space. In particular, a displaced world-sheet is a distinct world-sheet. The surfaces of loop quantum gravity, on the other hand, branch, and are defined in a background independent manner over a space without metric, where only their topology, not their location, matters. The existence of some connection between the two pictures has been advocated in particular by Smolin [131]; and Ashoke Sen has recently introduced a notion of “string networks” into string theory, paralleling the step from loops to spin network in loop quantum gravity [132].[†]

In conclusion, I believe that string theory and loop quantum gravity do represent real progress. With respect to few years ago, we now do better understand what may cause black hole entropy, and what a quantized spacetime might be.

However, in my opinion it is a serious mistake to claim that this is knowledge we have acquired about nature. Contrary to what is too often claimed even to the large public, perhaps with damage to the credibility of the entire theoretical community, these are only very *tentative* theories, without, so far, a single piece of experiment support. For what we really know, they could be right or entirely wrong. What we really know at the fundamental physical level is only the standard model and general relativity, which, within their domains of validity have re-

[†]In “Blue Mars”, the last novel of the science-fiction Mars trilogy by Kim Stanley Robinson [133], the fundamental physics of the 23rd century is based on a merging between loop and string theories!

ceived continuous and spectacular experimental corroboration, month after month, in the last decades. The rest is, for the moment, tentative and speculative searching.

But is worthwhile, beautiful, fascinating searching, which might lead us to the next level of understanding nature.

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