“Incerto tempore, incertisque loci”: Can we compute the exact time at which a quantum measurement happens?

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Without addressing the measurement problem (i.e. *what* causes the wave function to “collapse”, or to “branch”, or a history to become realized, or a property to actualize), I discuss the problem of the *timing* of the quantum measurement: assuming that in an appropriate sense a measurement happens, *when* precisely does it happen? This question can be posed within most interpretations of quantum mechanics. By introducing the operator M, which measures whether or not the quantum measurement has happened, I suggest that, contrary to what is often claimed, quantum mechanics does provide a precise answer to this question, although a somewhat surprising one.

Quantum mechanics was not originally formulated as a general dynamical theory of the world, but as the theory of quantum microsystems interacting with classical macrosystems. In the original formulation of the theory, the interaction of a quantum microsystem S with a classical macrosystem O is described in terms of “quantum measurements”. If the macrosystem O interacts with the variable q of the microsystem S, and S is in a superposition of states with different values of q, then the macrosystem O “sees” only one of the values of q, and the interaction modifies the state of S by projecting it into a state with that value.

It was noticed early that this formulation raises certain difficulties, particularly if we want to view the theory as the general dynamical theory of anything, and not just microsystems in the laboratory. These difficulties are usually referred to as the “measurement problem” and they have motivated ingenious attempts to modify either quantum mechanics or its interpretation.

In this letter, I do not discuss the measurement problem. Rather, I discuss a secondary problem, which as far as I understand, appears in most (but not all) formulations of quantum mechanics. The problem I discuss is the determination of the precise time at which the quantum measurement happens. Let me illustrate, without any ambition of precision or completeness, how this problem manifests itself in some popular interpretations of the theory. In one interpretation, a system S has a wave function, which collapses during a measurement. Is the collapse of the wave function instantaneous? If so, when precisely does it happen? In another interpretation, the system is described in terms of its properties, or values of its dynamical variables. These become manifest, and take definite values when observed, in a measurement. Do properties become manifest suddenly? When precisely do they become manifest, in a realistic laboratory experiment? In another interpretation, the wave function never collapses but it branches. When precisely does the branching happens? In another interpretation, the wave function never branches, but we, observers, navigate through it and sit in one or the other of its components. When is it, precisely, that the selection of this or that component happens? In another interpretation, there is no wave function, but only probabilities assigned to sequences of (quantum) events. When is it precisely that a quantum event “happens”? Namely, when precisely can we replace the statement “this may happen with probability p” with the statement “this has happened”?

The most common answer to these questions is that quantum mechanics does not determine the time at which the measurement happens. As is well known, for instance, von Neumann, pointed out that we can freely move the boundary between the quantum system and the observer, and therefore also shift the measurement’s presumed time, without affecting physical predictions [1]; and Heisenberg insisted that we can compute probability transitions between initial and final states, but we must abstain from asking “in between” questions [2]. According to this view, the question “when does the measurement happen” does not seem to be a well posed question.

In this letter, I claim that, contrary to that view, quantum mechanics *does* give a precise answer to this question, although a peculiar answer. More precisely, I claim that: (i) a precise (operational) sense can be given to the question of the timing of the measurement; (ii) we can then compute the time at which the measurement happens (in the sense specified) using standard quantum techniques; (iii) the (more or less realistic) interpretation of the physical meaning of this time is no more problematic than the interpretation of any other quantum result. I will also venture in suggesting that this observation might partially illuminate some aspects of the major problem – the measurement problem itself.

The answer I present here is based on two main ideas. First, that the question “When does the measurement happen?” is quantum mechanical in nature, and not classical. Therefore its answer must be probabilistic. In general, in quantum mechanics the answer to a question such as “Is the spin up?” is not “Yes” or “No”, but rather:

"Yes with probability, say, 1/2", which implies that the spin will come out "up" in half the repetitions of the experiment. Similarly, I am roughly going to argue that "half way through a measurement" the measurement is not "partially happened", but just "happened with probability 1/2", or "already realized in half of the repetitions of the experiment". The second idea is that the question "When does the measurement happen?" does not regard the measured quantum system S alone, but rather the coupled system formed by the observed system S and the observer system O. Therefore the appropriate theoretical setting for answering this question is the quantum theory of the two coupled systems. In particular, I will submit that there is no contradiction in using the quantum theory of S for describing the observed behavior of S and the quantum theory of the coupled $S-O$ system for answering the question of the precise timing of the measurement.

More in detail, this paper is based on a technical observation: there is an operator that has a natural interpretation as measuring whether or not the measurement has happened. I will denote this operator as M (for Measurement). The operator M is a projection operator, with the eigenvalue 1 meaning "the measurement has happened", and the eigenvalue 0 meaning "the measurement has not happened". By applying standard quantum mechanical rules to *this* operator, at every time t we can compute a precise (although probabilistic) answer to the question whether or not the measurement has happened.

The description of the quantum measurement that I employ here is based on the Peres-Wootters analysis of the quantum measurements of finite duration [3]. Several of the ideas on which this paper is based were first introduced therein. In particular, the key idea that measurement can be defined operationally, in terms of the correlation that could be detected by a *second* external apparatus, is in [3]. Peres and Wootters do not introduce the operator M considered here, but discuss the related idea of computing the probability distribution of the measured variable, conditionalized by the observed position of the apparatus' pointer. The aim of Peres and Wootters was to provide a physical analysis of realistic measurements and their intrinsic limitations. Here, on the other hand, I am interested in the general notion of measurement and on what we can say on *when* we replace the statement "this may happens with probability p" with the statement "this has happened".

The result described here has emerged within the relational interpretation of quantum mechanics [4]. See also the strictly related Kochen's interpretation [5], and reference [6], where these two interpretations are compared. This result, however, does not require such interpretations, and I present it here in a form which is independent from the interpretation of quantum mechanics one holds.

Just to fix a language, I will refer here to a traditional (and a bit out of fashion) "wave function collapse" terminology, leaving to the reader the burden of translating what I say in his or her preferred language. Consider a physical system S (an electron). Assume that S interacts with another physical system O (an apparatus measuring the spin of the electron), and that the interaction between S and O qualifies as a quantum measurement of the variable q of the system S. Choosing the simplest setting, and following a terminology which is now standard, I assume that q has only two discrete eigenvalues, a and b, and denote by $|a\rangle$ and $|b\rangle$ the two corresponding eigenstates. The interaction between S and O is governed by an interaction hamiltonian H_I. A necessary condition for the interaction to be a measurement is that we can prepare O in an initial state, which we denote $|init\rangle$, such that in a finite time T the interaction will evolve (perhaps up to some approximation) $|a\rangle \otimes |init\rangle$ into $|a\rangle \otimes |Oa\rangle$ and $|b\rangle \otimes |init\rangle$ into $|b\rangle \otimes |Ob\rangle$, where $|Oa\rangle$ and $|Ob\rangle$ are states of O that we can identify as "the pointer of the apparatus indicates that q has value a", or b, respectively. Up to now, I have only given definitions. As it is well known, the linearity of quantum mechanics implies that if S is initially in a quantum superposition $c_a|a\rangle + c_b|b\rangle$, the combined system will evolve from

$$\Psi(0) = \big(c_a|a\rangle + c_b|b\rangle\big) \otimes |init\rangle \tag{1}$$

into

$$\Psi(T) = c_a|a\rangle \otimes |Oa\rangle + c_b|b\rangle \otimes |Ob\rangle. \tag{2}$$

Examples of models of interactions of the kind described are well known (see for instance [7]), and are easy to construct in the case of simple experiments such as a Stern-Gerlach measurement.*

For some reason, at some point we have to (or we can) replace the pure state $\Psi(T)$ with a mixed state. Equivalently, we replace $\Psi(T)$ with either

$$\Psi_a = |a\rangle \otimes |Oa\rangle, \tag{3}$$

or

$$\Psi_b = |b\rangle \otimes |Ob\rangle, \tag{4}$$

where, of course, the probability of having one or the other is $|c_a|^2$ and $|c_b|^2$ respectively. Since here I am not discussing the measurement problem, I will not address the problem of the meaning of this step (whether it is

*The difficulty of describing the dynamics of quantum measurement in terms of Schrödinger evolution is not in showing how the system may evolve into a correlated state. It is in understanding how it may go from the correlated state to a mixture, or to an eigenstate of the measured quantity.

a physical event, a change in our knowledge, a mental event, a perspectival event, or other). Nor I will discuss whether, or how, or why, or under which circumstances, this wave function collapse happens.

Rather, I simply assume that in some appropriate sense the wave function changes from (2) to either (3) or (4) and the quantity q acquires a definite value; and I focus on the question of what we can say about the precise time t at which this happens.

At the time T, the system has reached the state (2), and a definite correlation between the pointer variable, with eigenstates $|Oa\rangle$ and $|Ob\rangle$, and the system variable, with eigenstates $|a\rangle$ and $|b\rangle$, is established. Before that time, in general, this correlation is absent, or incomplete. If the wave function has collapsed, and the state is either (3) or (4), the correlation is present as well. In other words, the happening of the measurement is tied to the fact that the complete correlation is established.

This well known observation has prompted many to suggest that the measurement (of the quantity q, by the pointer variable) happens in the moment in which the complete correlation is established. In modal interpretations, for instance, the quantity q "has value" only if the state is in the (Schmidt, or biorthogonal) form (2). This would lead us to say that the measurement of q happens instantaneously in the moment the complete correlation is established.[†] I consider here a different approach to the problem.

Notice that there is a precise sense in which we can *measure* whether, at some time t intermediate between $t = 0$ and $t = T$, the measurement has happened or not. In fact, let us suppose that at t we bring a further external apparatus O' in and we measure the value of q as well as the position of the pointer (the two commute, of course). Then, either (*Case 1*) we find the pointer in the correct position corresponding to the value of q that we have found - namely we obtain $\{a$ and $Qa\}$, or $\{b$ and $Qb\}$; or (*Case 2*) the pointer is not on the correct position.[‡] The question "Has the pointer already measured q at time t?" has therefore a natural interpretation: if we find *Case 1*, we can say that the pointer has already measured q; if we find *Case 2*, it hasn't. Thus, the question of the timing at which a measurement happens can be given a simple operational interpretation, independent from any metaphysics, or any interpretation of quantum mechanics, one might hold. The operational procedure described assigns a precise meaning to the question. Can we then answer the question? Certainly yes, because quantum mechanics provides us the tool for predicting the outcome of the measurement described. More precisely, it provides us the tool for computing the probabilities of finding *Case 1* or *Case 2*, for any given state $\Psi(t)$ of the combined $S - O$ system. These probabilities are obtained as follows.

Define the operator M on the state space of the $S - O$ system, as the projection operator on the subspace spanned by the two states Ψ_a and Ψ_b:

$$M \equiv |\Psi_a\rangle\langle\Psi_a| + |\Psi_b\rangle\langle\Psi_b|. \tag{5}$$

Namely

$$M\big(|a\rangle \otimes |Oa\rangle\big) = |a\rangle \otimes |Oa\rangle \tag{6}$$
$$M\big(|b\rangle \otimes |Ob\rangle\big) = |b\rangle \otimes |Ob\rangle \tag{7}$$
$$M\big(|a\rangle \otimes |Ob\rangle\big) = 0 \tag{8}$$
$$M\big(|b\rangle \otimes |Oa\rangle\big) = 0 \tag{9}$$

and

$$M\big(|\psi\rangle \otimes |\phi\rangle\big) = 0 \tag{10}$$

if $\langle\phi|Oa\rangle = 0$ and $\langle\phi|Ob\rangle = 0$.

M is a self-adjoint operator on the Hilbert space of the coupled system. Therefore it may admit an interpretation as an observable property of the coupled system. In all the eigenstates of M with eigenvalue 1 the pointer variable correctly indicates the value of q. In all the eigenstates of M with eigenvalue 0, it does not. Therefore, M has the following interpretation: $M = 1$ means that the pointer (correctly) measures q. $M = 0$ means that it does not. Now, when the pointer of the apparatus correctly measures the value of the observed quantity, we say that the measurement has happened. Therefore we can say that $M = 1$ has the physical interpretation "The measurement has happened", and $M = 0$ has the physical interpretation "the measurement has not happened".

Since M is a genuine self-adjoint operator on the Hilbert space of the coupled system, and since it has an unambiguous physical interpretation, we can apply the standard interpretation rules of quantum mechanics to it. In particular, for all states that are not eigenstates of M, quantum mechanics tells us that we must not interpret these states as having "intermediate" values; but, rather, they have one or the other values with a certain probability. That is, if a state Ψ of the coupled system is

[†]This solution does not convince me for various reasons. First, nature is never clean, and in a realistic case the exact form (2) is never attained. One might reply that if the state is "close" to the form (2), then it will still have a Schmidt decomposition into a basis "close" to the eigenbasis of q; but unfortunately this is not true: Schmidt bases jump badly anytime the state passes close to a degenerate state, and a state "very close to (2)" may defines a Schmidt basis in which a quantity completely unrelated to q is diagonal. Second, quantum mechanics gives us probabilistic statements about nature. Why should it give us a sharp answer for the measurement time?

[‡]Typically, the pointer will fail to be in the correct position not because it is in the wrong position, but because it is still in $|init\rangle$.

not a perfectly correlated state of the form (2), (3), or (4), then in general we must conclude that Ψ is a superposition of a state in which the measurement has happened and a state in which the measurement has not happened.

In particular, if we follow the Schrödinger evolution $\Psi(t)$ of the state of the coupled system from $t = 0$ to $t = T$, then at every intermediate t we can compute the probability $P(t)$ that the measurement has happened

$$P(t) = \langle \Psi(t) | M | \Psi(t) \rangle \tag{11}$$

For a good measurement, $P(t)$ will be a smooth function that goes monotonically from 0 to 1 in the time interval 0 to T. Therefore, half way through the measurement, we cannot say that the measurement has "half happened": we must say that the measurement "has happened with probability $P(t)$".

The last statement has a precise operational meaning: we may repeat the physical process in which O measures the quantity q of S a large number of trials. In each trial, at a time $t < T$ after the beginning of the process, we can check whether or not the measurement has already happened by having the second "external" apparatus O' measuring q and the position of the pointer. If the two match, then O has detected the "correct" value of q: "a measurement has happened". Standard quantum mechanical arguments show that $P(T)$, defined in (11), gives precisely the fraction of the trials in which we will detect that the measurement has happened.

The correctness of this interpretation of M is particularly evident in the following case. Imagine that the two pointer states $|Oa\rangle$ and $|Ob\rangle$ represent two LED's that light up when the O apparatus has measured a, or, respectively, b. Also, assume that in the measurement the "wrong" states $|a\rangle \otimes |Ob\rangle$ and $|b\rangle \otimes |Oa\rangle$ happens with null or negligible probability (a realistic assumption for a good measurement). If we ask an experimentalist when is the measurement completed, he will probably answer: "when one of the LED's lights up". But the distribution probability of the time at which the LED lights up can be theoretically computed, and is given precisely by $P(t)$, because M is the projector on the subspace on which one of the LED's is on.

If $P(t)$ is the probability that the measurement has happened at time t, then the probability (density) $p(t)$ that the measurement happens between time t and time $t + dt$ is

$$\begin{aligned} p(t) &= \frac{dP(t)}{dt} \\ &= \frac{d}{dt} \langle \Psi(t) | M | \Psi(t) \rangle \\ &= \langle \Psi(t) | [M, H] | \Psi(t) \rangle, \end{aligned} \tag{12}$$

where H is the total hamiltonian. I introduce the operator

$$m = [M, H]. \tag{13}$$

Its expectation value at time t gives the probability density for the measurement to happen at time t

$$p(t) = \langle \Psi(t) | m | \Psi(t) \rangle. \tag{14}$$

For a good measurement in which $P(t)$ grows smoothly and monotonically from zero to one, $p(t)$ will a "bell shaped" curve, defining the time at which the measurement happens, and its quantum dispersion.

The operators M and m can be generalized to an arbitrary number of eigenvalues of the quantity measured. If the apparatus is able to distinguish the (eigen)values $a_1 \ldots a_n$ of the variable q, by means of the pointer positions $Oa_1 \ldots Oa_n$, then M is given by

$$M = \sum_{i=1,n} \big(|a_i\rangle \otimes |Oa_i\rangle \big) \big(\langle a_i| \otimes \langle Oa_i| \big), \tag{15}$$

and this definition can easily be generalized to continuous spectra.

Thus, we can conclude that quantum mechanics provides a precise meaning and a precise answer to the question of when a measurement happens. The answer is, like any other answer to physical questions in quantum theory, a probabilistic one.[§] Using the operator m defined in (13,15), the probability $p(t)$ that the measurement happens at time t can be computed at every intermediate time from (14). This quantity can be explicitly computed from the standard quantum physics of the system.

I close with some observations and general comments. The first observation refers to the relation between the issue discussed here and the general measurement problem. The interpretation of the operator M and of the predictions it yields are infested by the same subtle problems that infest any other quantum operator. For instance, one may ask whether M measures "if the measurement has indeed really happened", or whether "we shall find that the measurement has happened if we check". The distinction refers to the very possibility of assigning values to quantities irrespectively from observations, and thus touches the core of the measurement problem. As I said, I do not discuss this major conundrum here. The interpretation of the time at which a measurement happens has no less and no more difficulties than the interpretation any other quantity of a quantum system. In particular, I warn the reader of not charging my claim with more

[§]Quantum events, the individual events at the basis of modern physics, happen randomly – precisely as the Democritean $\kappa\lambda\iota\nu\grave{\alpha}\mu\epsilon\nu$, the individual events at the basis of ancient physics, which, in Lucretius marvelous verses, happen "incerto tempore . . . incertisque loco": at a random time and in a random place.[8]

meaning than what it deserves. I am not claiming that there is an "element of reality" in the fact that a measurement has happened, or that, in general "measurement having happened or not" is an *objective property* of the coupled system. I am only claiming: i) that the the time t at which a measurement happens is a well defined concept in the theory (so precise that it can be defined operationally); and ii) that this time, or, more precisely its probability distribution, can be explicitly computed if the unitary dynamics of the system-apparatus dynamics is known. Thus, this time is as well defined a quantity as, say, the position of a particle: if we know the dynamics, we can, at any moment, compute its probability distribution.

The notion of measurement that I have referred to in this paper does not require that phase coherence is definitely lost. Also, I have made no reference to physical decoherence. If quantum mechanics is *exactly* correct in nature (as I assume here), then phase coherence is never definitely lost. (In principle one can always make an interference experiment between the spin up and the spin down components of the state, even after a macroscopic Stern Gerlach apparatus has registered the outcome on printed paper.) In practice, phase coherence is lost via physical decoherence, say into the environment, as beautifully realized by Zurek [9], making the pure and the mixed states effectively indistinguishable. Thus, the question of when a quantum correlation (the pure state) can be *effectively* described as a mixed state is already solved. This, however, is not the question I have addressed here. The question I have addressed refers to the fact that one or the other of the physical values of q become, at some point, part of our reality: a mystery that, as Zurek emphasizes in the conclusion of [9], remains also after understanding physical decoherence. Without addressing the issue of how this actualization of the values of q may come about, I have discussed here "when" we may say it happens.

An issue often discussed in the context of the measurement problem is the one of imperfect measurements. Imagine that an interaction between a system S and an "apparatus" O is such that after the interaction the coupled system ends up in a state which is close, but not exactly equal, to the state (2). Has the measurement happened, even if perfect correlation has not been established? The approach presented here offers a solution to the conundrum. The solution is that the measurement has happened with high probability. In practical terms, this means that it has happened in almost all, but not all, the trials.

A related observation is that the existence of the M operator might perhaps cause an embarrassment for the modal interpretation. In fact, quantum mechanics contains an operator corresponding to the question "When does q take a definite value?", a question addressed in detail within modal views. But the answer that quantum mechanics provides seems inconsistent with the modal view that q has a definite value only when the requisite biorthogonal correlation is established.**

A possible objection to the present work is that it is relevant within certain interpretations of quantum mechanics, but it is irrelevant within the interpretations in which no mention of quantum measurement is made, such as the histories interpretations [10]. This is a delicate issue, which requires a case by case discussion that would not fit into this brief note. I only sketch here some general considerations on this regard. Following [3], I have given an operational definition of the measurement's timing. Under such a definition, measurement's timing makes sense within any interpretation. But why would we want to discuss measurement within a measurement-free interpretation? As suggested in the beginning of the paper, I think that within most, if not all, interpretations, there is always a point in which we have to jump from the statement that something *may* happen with probability p, to the statement that something, in some appropriate sense, *has* happened. Otherwise, what is it that distinguishes the (probabilistic) *predictions* we *make* from the (non-probabilistic) *data* we *have* about the world, on which those predictions are based? As cleanly made clear in particular by the histories interpretation, quantum mechanical predictions do not depend just on the data: they also depend on the question asked. One can refuse to discuss the step in which predictions become data, as prescribed in the histories interpretation, and reset the theoretical machinery at every newly acquired data. This is fine. But nothing then prevents somebody else to ask the further question of whether one can predict if and when the theoretical machinery can be reset. The histories-interpretation view, I think, is that this question has no answer within quantum mechanics. This is precisely Heisenberg's claim, mentioned above. Here, contrary to that claim, I have suggested that one can still keep the interpretation as it is, but *add* to it, coherently, a meaningful prediction on when the resetting of the theoretical machinery can be made.††

**I thank John Earman for this observation.

†† I add a more speculative remark. I suspect that in spite of brilliant attempts to purge quantum mechanics of any "special moment" in time in which measurements happen, such "special moments" remain in most, if not all, interpretations of quantum mechanics, sometimes in a hidden fashion. A coherent way of avoiding this problem is to relegate it to a still to be discovered theory of consciousness, as David Mermin has recently, coherently, suggested in [11] (on a related vein, see [12]). Short of this noble and courageous declaration of failure, we may purge quantum mechanics from the expression "measurement", pleasing Bell [13], but the mystery posed by these "special moments", whether we call them

The answer to the timing problem I have discussed is peculiar in one respect: it involves the simultaneous use of the formal apparatus of quantum mechanics at two levels. I have combined the use of the quantum theory of the $S-O$ system, in which the operator M is defined, with the quantum theory of the system S alone, in which we may talk of the collapse of the wave function due to the interaction of S with O. There is something slightly conceptually anomalous in doing that. I think that this anomaly is not a defect of the idea proposed here. Rather, it represents its essential aspect. This procedure might shed some light on the relational aspects of quantum theory. There is a subtle point here: the fact that a "time of measurement" can be computed does not imply that the wave function collapse is an objective observer-independent event. On the contrary, to my understanding, the core of the measurement problem is the fact that if we take quantum mechanics as a general theory of mechanics, we must combine two apparently badly contradictory facts. On the one hand, the value of the variable q is definitely directly observed by O (by us). On the other hand, phase coherence is never really lost, and thus the "other branch" is still somehow there, even if not *for* O (for us). The precise time at which the measurement happens can be observed, and can be operationally defined in the manner described above, but not by O. Rather, by the third system (O') making measurements on the $S-O$ system. Simultaneous use of these different levels does not lead to any contradiction [4, 5]. And there is no regress at infinity, unless we inquire about absolute reality, an inquiry which, I suspect, is illegitimate. These ideas are discussed, in various versions, in [5] and in [4, 6]. The existence of the M operator and the solution of the timing problem suggested here find a natural framework within these views, but do not require them.

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measurements, quantum events, or otherwise, or we refuse to name them, remains. Talking about their timing, as I have attempted here, is an indirect way of addressing the issue.

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